Trig Identities Which Will Need To Be Recalled or Derived Rapidly:

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$1 - \cos^2 x = \sin^2 x,$$
$$1 - \sin^2 x = \cos^2 x,$$

$$\boxed{1 + \tan^2 x = \sec^2 x}$$

$$\sec^2 x - 1 = \tan^2 x,$$
$$\sec^2 x - \tan^2 x = 1,$$

$$\boxed{\cot^2 x + 1 = \operatorname{cosec}^2 x}$$

$$\operatorname{cosec}^2 x - 1 = \cot^2 x,$$
$$\operatorname{cosec}^2 x - \cot^2 x = 1.$$

$$\boxed{\sin(a+b) = \sin a \cos b + \cos a \sin b}$$

$$\boxed{\sin 2a = 2 \sin a \cos a}$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\boxed{\cos(a+b) = \cos a \cos b - \sin a \sin b}$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\boxed{\cos 2a = \cos^2 a - \sin^2 a}$$

$$\boxed{\cos 2a = 2 \cos^2 a - 1}$$

$$\boxed{\cos^2 a = \frac{1 + \cos 2a}{2}}$$

$$\boxed{\cos 2a = 1 - 2 \sin^2 a}$$

$$\boxed{\sin^2 a = \frac{1 - \cos 2a}{2}}$$

Averaging

$$\begin{cases} \sin a\cos b+\cos a\sin b=\sin(a+b) \\ \sin a\cos b-\cos a\sin b=\sin(a-b) \end{cases}$$

we obtain

$$\boxed{\sin a\cos b=\tfrac{1}{2}\Big(\sin(a+b)+\sin(a-b)\Big)}$$

Averaging

$$\begin{cases} \cos a\cos b-\sin a\sin b=\cos(a+b) \\ \cos a\cos b+\sin a\sin b=\cos(a-b) \end{cases}$$

we obtain

$$\boxed{\cos a\cos b=\tfrac{1}{2}\Big(\cos(a+b)+\cos(a-b)\Big)},$$

$$\boxed{\sin a\sin b=\tfrac{1}{2}\Big(\cos(a+b)-\cos(a-b)\Big)}.$$

Dividing $\left\{ \begin{aligned} \sin(a+b) &= \sin a\cos b + \cos a\sin b \\ \cos(a+b) &= \cos a\cos b - \sin a\sin b \end{aligned} \right.$ **we obtain**

$$\tan(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a\cos b + \cos a\sin b}{\cos a\cos b - \sin a\sin b}$$

$$= \frac{\dfrac{\sin a\cos b}{\cos a\cos b} + \dfrac{\cos a\sin b}{\cos a\cos b}}{\dfrac{\cos a\cos b}{\cos a\cos b} - \dfrac{\sin a\sin b}{\cos a\cos b}} = \frac{\dfrac{\sin a}{\cos a} + \dfrac{\sin b}{\cos b}}{1 - \dfrac{\sin a}{\cos a}\cdot\dfrac{\sin b}{\cos b}},$$ **so that**

$$\boxed{\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}}$$

Using $A = \tan(\arctan A)$ and $B = \tan(\arctan B)$ we have

$$\tan\left(\arctan A + \arctan B\right) = \frac{\tan\left(\arctan A\right) + \tan\left(\arctan B\right)}{1 - \tan\left(\arctan A\right)\tan\left(\arctan B\right)}$$

$$= \frac{A+B}{1-AB}, \text{ so that}$$

$$\arctan A + \arctan B = \begin{cases} \arctan\dfrac{A+B}{1-AB} & \text{if } AB < 1 \\ \arctan\dfrac{A+B}{1-AB} + \pi & \text{if } AB > 1, A > 0, B > 0 \\ \arctan\dfrac{A+B}{1-AB} - \pi & \text{if } AB > 1, A < 0, B < 0 \end{cases}$$

More Trig Identities:

$$\tan u = \frac{\sin u}{\cos u} = \frac{2\sin u \cos u}{2\cos^2 u} = \frac{2\sin u \cos u}{(2\cos^2 u - 1) + 1} = \frac{\sin 2u}{\cos 2u + 1},$$

which becomes, when u is replaced by $\frac{x}{2}$, $\boxed{\tan\frac{x}{2} = \frac{\sin x}{\cos x + 1}}$

$$\tan u = \frac{\sin u}{\cos u} = \frac{2\sin^2 u}{2\sin u \cos u} = \frac{(2\sin^2 u - 1) + 1}{2\sin u \cos u} = \frac{1 - \cos 2u}{\sin 2u},$$

which yields similarly $\boxed{\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x} = \operatorname{cosec} x - \cot x}$

$$\left(\textbf{Check: } (1 - \cos x)(1 + \cos x) = 1 - \cos^2 x = \sin^2 x\right)$$

More Identities:

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\tan\frac{\left(x+\frac{\pi}{2}\right)}{2}=\frac{\sin\left(x+\frac{\pi}{2}\right)}{\cos\left(x+\frac{\pi}{2}\right)+1}$$

$$=\frac{\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}}{\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}+1}=\frac{\sin x\cdot 0+\cos x\cdot 1}{\cos x\cdot 0-\sin x\cdot 1+1},$$

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\tan\frac{\left(x+\frac{\pi}{2}\right)}{2}=\frac{1-\cos\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)}$$

$$=\frac{1-\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}}{\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}}=\frac{1-\cos x\cdot 0+\sin x\cdot 1}{\sin x\cdot 0+\cos x\cdot 1},$$

$$\boxed{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{\cos x}{1-\sin x}=\frac{1+\sin x}{\cos x}}$$

Another Derivation:

$$\tan\left(u+\frac{\pi}{4}\right)=\frac{\sin\left(u+\frac{\pi}{4}\right)}{\cos\left(u+\frac{\pi}{4}\right)}=\frac{\sin u\cos\frac{\pi}{4}+\cos u\sin\frac{\pi}{4}}{\cos u\cos\frac{\pi}{4}-\sin u\sin\frac{\pi}{4}}$$

$$=\frac{\sin u\cdot\frac{1}{\sqrt{2}}+\cos u\cdot\frac{1}{\sqrt{2}}}{\cos u\cdot\frac{1}{\sqrt{2}}-\sin u\cdot\frac{1}{\sqrt{2}}}=\frac{\sin u+\cos u}{\cos u-\sin u}\cdot\frac{\sin u+\cos u}{\cos u+\sin u}$$

$$=\frac{\sin^2 u+2\sin u\cos u+\cos^2 u}{\cos^2 u-\sin^2 u}=\frac{1+\sin 2u}{\cos 2u}$$

similarly becomes, when u is replaced by $\frac{x}{2}$,

$$\boxed{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{1+\sin x}{\cos x}}$$