$$\begin{aligned}
\textbf{Now a specific way to express } \sin h &= h + o(h), \quad \text{as } h \to 0,\\
\textbf{in the format: } f(x+h) &= f(x) + f'(x)h + o(h)\\
\textbf{is } \sin h &= 0 + 1 \cdot h + o(h),\\
\textbf{which yields } \sin 0 = f(0) &= 0\\
\textbf{and } \sin' 0 = f'(0) &= 1,
\end{aligned}$$

$$\begin{aligned}
\textbf{and a specific way to express } \cos h &= 1 + o(h), \quad \text{as } h \to 0,\\
\textbf{in the format: } f(x+h) &= f(x) + f'(x)h + o(h)\\
\textbf{is } \cos h &= 1 + 0 \cdot h + o(h),\\
\textbf{which yields } \cos 0 = f(0) &= 1\\
\textbf{and } \cos' 0 = f'(0) &= 0.
\end{aligned}$$

Using $\sin h = h + o(h)$, and

$\cos h = 1 + o(h)$, as $h \longrightarrow 0$,

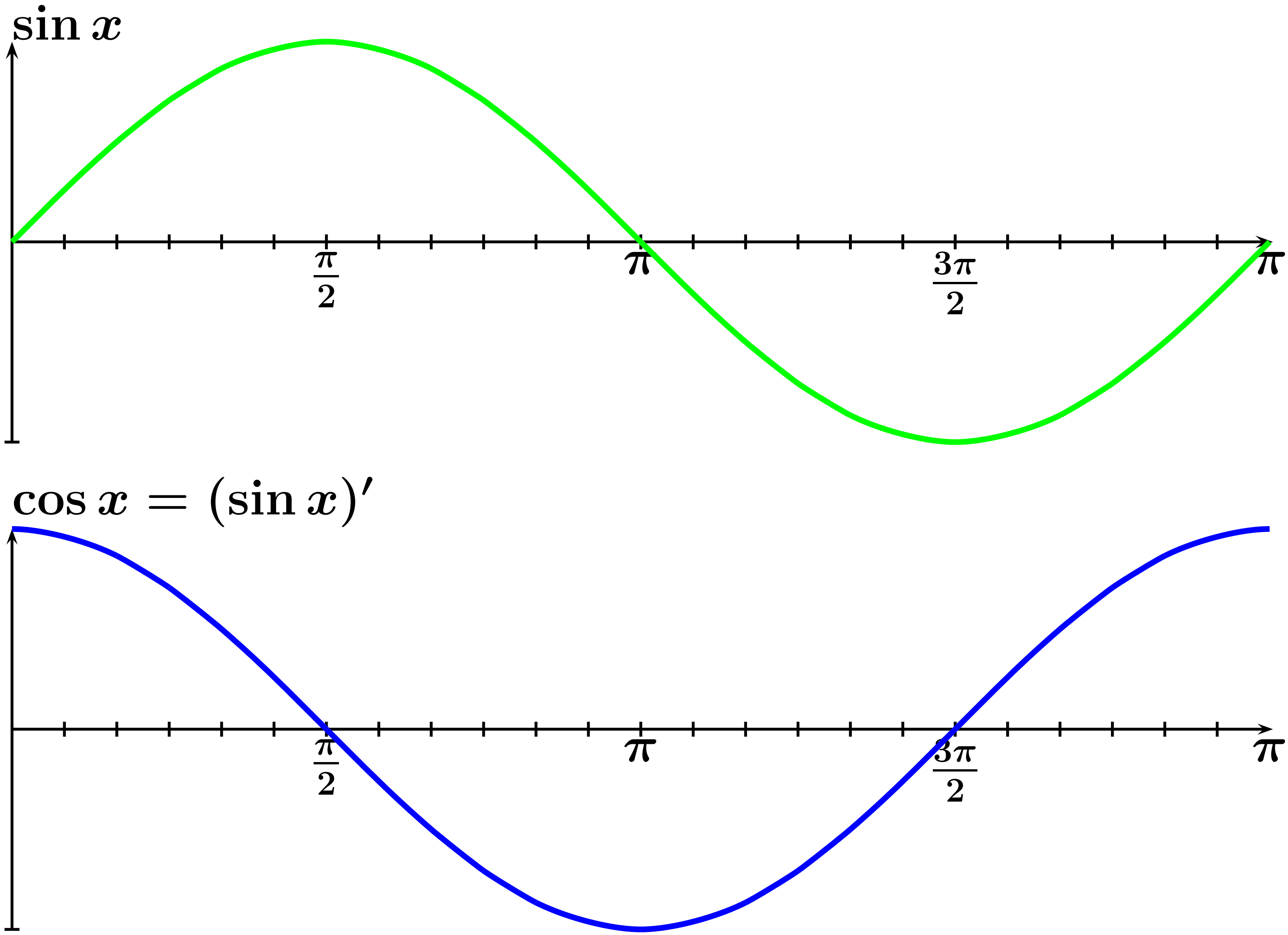
we can expand

$$
\begin{aligned}
\sin(x+h) &= \sin x \cos h + \cos x \sin h \\
&= \sin x\bigl(1 + o(h)\bigr) + \cos x\bigl(h + o(h)\bigr) \\
&= \sin x \qquad\qquad + o(h)\sin x \\
&\qquad + h\cos x + o(h)\cos x \\
&= \sin x \quad + h\cos x + o(h),
\end{aligned}
$$

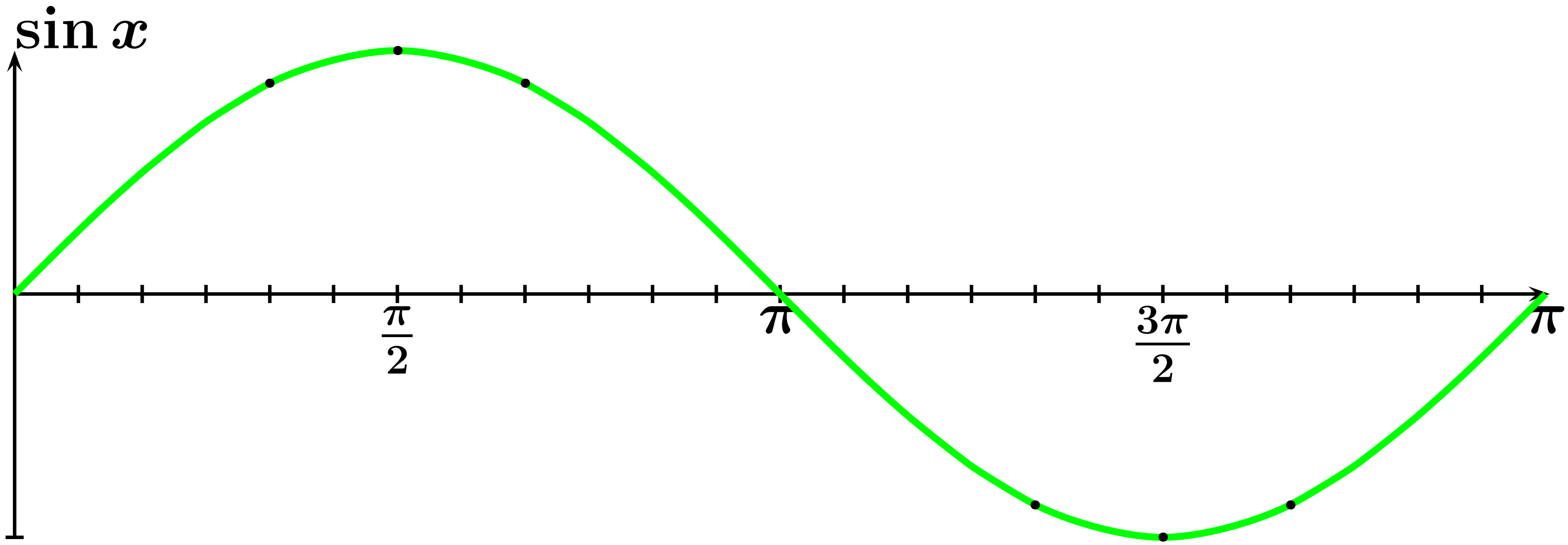
so that $\boxed{\dfrac{d}{dx}\sin x = \cos x}$

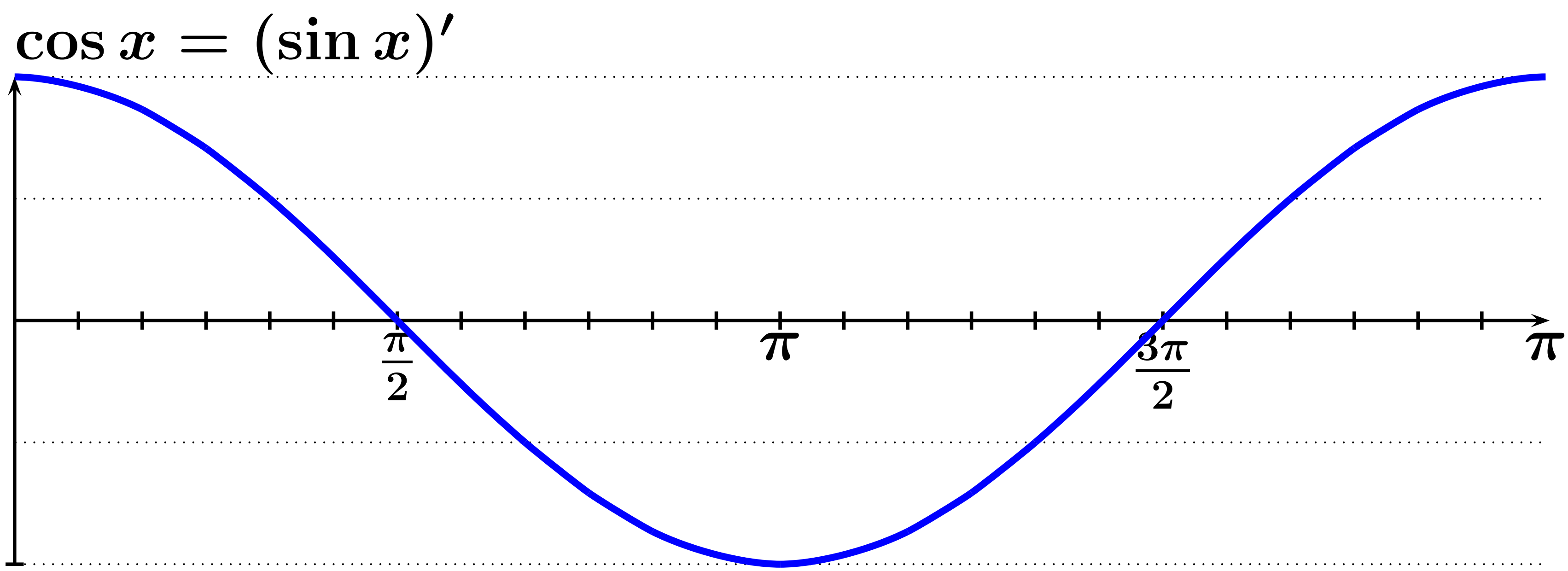
**The traditional way to get the derivative of $\sin x$:**

$$\begin{aligned}
&\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\
= &\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}\\
= &\lim_{h\to 0}\frac{\cos x\sin h+\sin x(\cos h-1)}{h}\\
= &\lim_{h\to 0}\left(\cos x\cdot\frac{\sin h}{h}+\sin x\cdot\frac{\cos h-1}{h}\right)\\
&= \cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}+\sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}\\
&= \cos x\cdot 1+\sin x\cdot 0\\
&= \cos x
\end{aligned}$$



sin x
π/2
π
3π/2
π
cos x = (sin x)′
π/2
π
3π/2
π


sin x
π/2
π
3π/2
π


cos x = (sin x)′
π/2
π
3π/2
π

Using $\sin h = \; h + o(h),$ and

$\cos h = \; 1 + o(h), \quad$ as $h \to 0,$

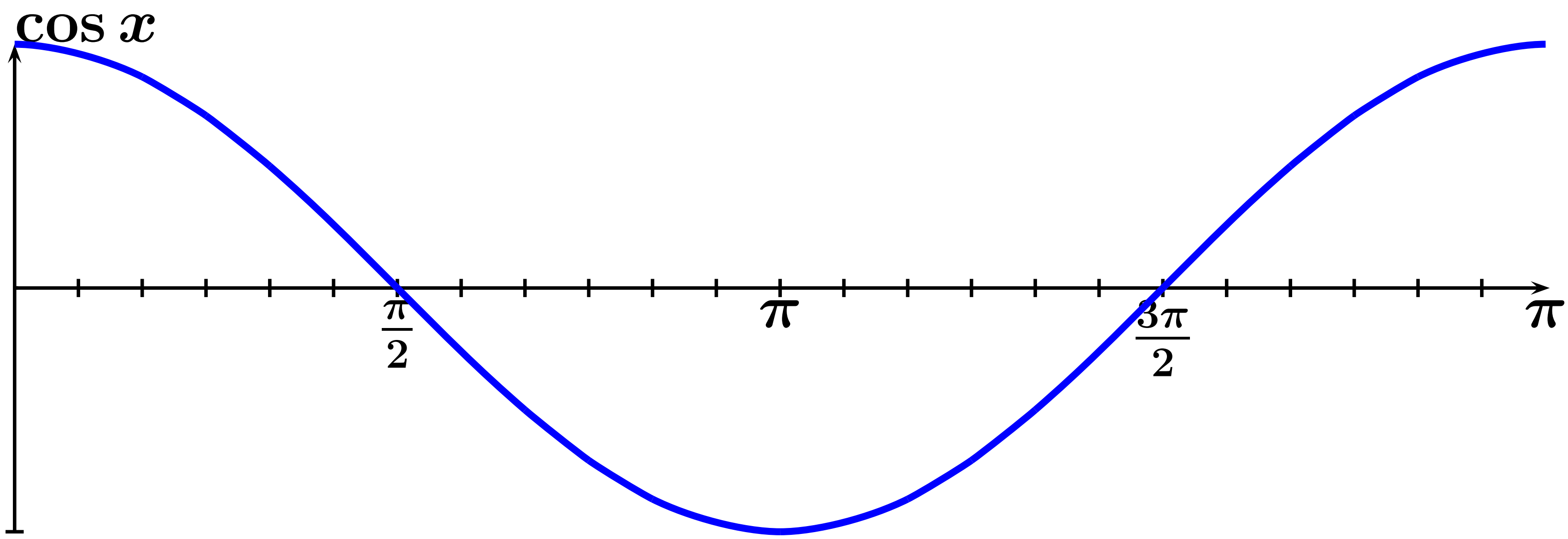
we can expand

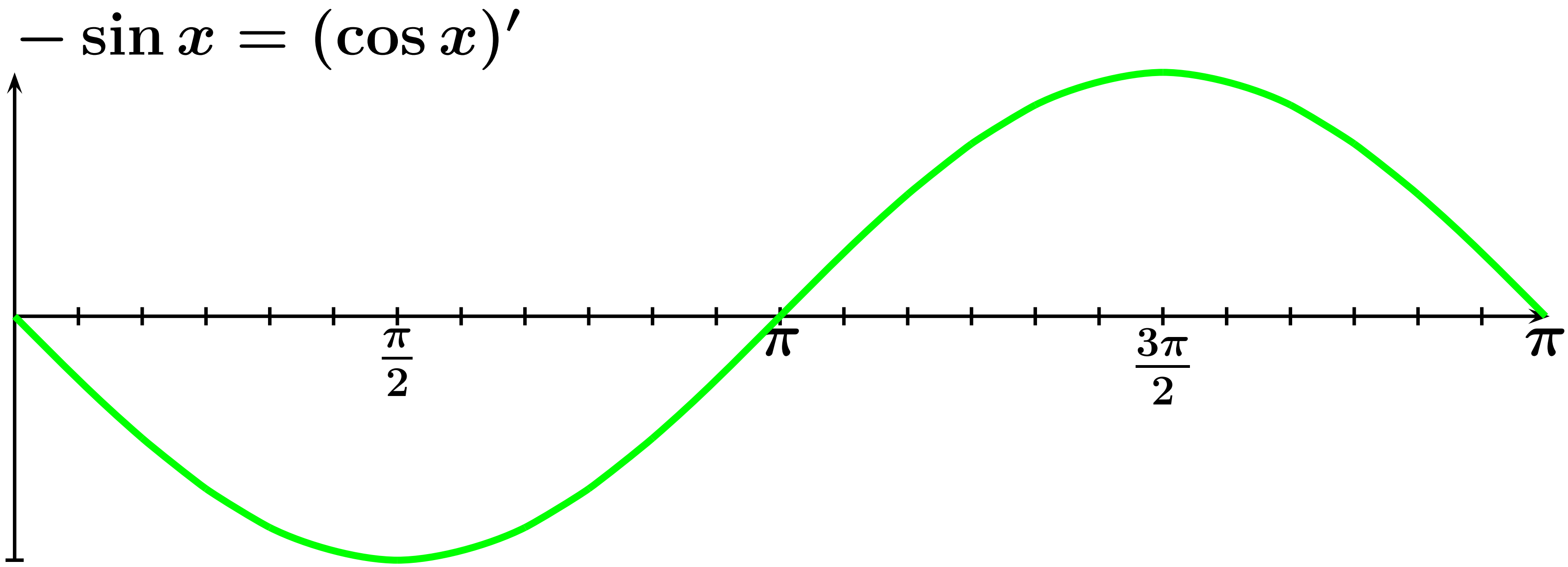
$$
\begin{aligned}
\cos(x+h) &= \cos x \cos h - \sin x \sin h \\
&= \cos x \big(1 + o(h)\big) - \sin x \big(h + o(h)\big) \\
&= \cos x \qquad\qquad\qquad + o(h) \cos x \\
&\qquad\quad - h \sin x - o(h) \sin x \\
&= \cos x \quad - h \sin x + o(h),
\end{aligned}
$$

so that $\boxed{\dfrac{d}{dx} \cos x = -\sin x.}$

**The traditional way to get the derivative of $\cos x$:**

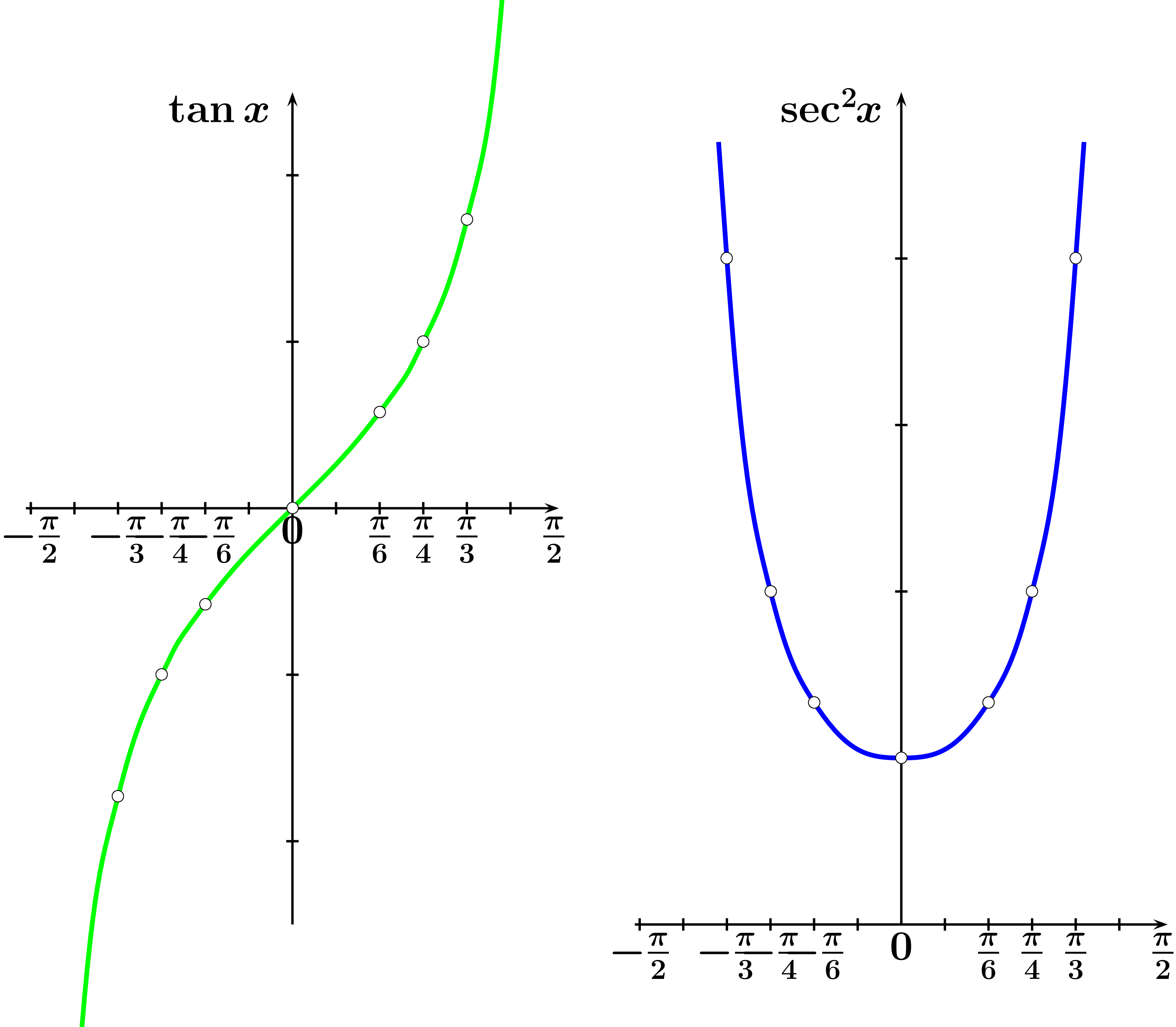
$$
\begin{aligned}
& \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} \\
= & \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\
= & \lim_{h\to 0} \frac{-\sin x \sin h + \cos x(\cos h - 1)}{h} \\
= & \lim_{h\to 0} \left( -\sin x \cdot \frac{\sin h}{h} + \cos x \cdot \frac{\cos h - 1}{h} \right) \\
= & -\sin x \cdot \lim_{h\to 0} \frac{\sin h}{h} + \cos x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} \\
= & -\sin x \cdot 1 + \cos x \cdot 0 \\
= & -\sin x
\end{aligned}
$$


$\cos x$
$\frac{\pi}{2}$
$\pi$
$\frac{3\pi}{2}$
$\pi$


$-\sin x = (\cos x)'$
$\frac{\pi}{2}$
$\pi$
$\frac{3\pi}{2}$
$\pi$

$$\begin{aligned}\frac{d}{dx}\tan x &= \frac{d}{dx}\ \frac{\sin x}{\cos x}\\ &= \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}\\ &= \frac{(\cos x)\cos x - \sin x(-\sin x)}{\cos^2 x}\\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x}\\ &= \sec^2 x\end{aligned}$$

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$$



$\tan x$
$-\frac{\pi}{2}$ $-\frac{\pi}{3}$ $-\frac{\pi}{4}$ $-\frac{\pi}{6}$ 0 $\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{\pi}{2}$
$\sec^2 x$
$-\frac{\pi}{2}$ $-\frac{\pi}{3}$ $-\frac{\pi}{4}$ $-\frac{\pi}{6}$ 0 $\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{\pi}{2}$

$$
\begin{aligned}
\frac{d}{dx}\cot x &= \frac{d}{dx}\;\frac{\cos x}{\sin x} \\
&= \frac{(\cos x)'\sin x - \cos x(\sin x)'}{\sin^2 x} \\
&= \frac{(-\sin x)\sin x - \cos x(\cos x)}{\sin^2 x} \\
&= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\
&= \frac{-1}{\sin^2 x} \\
&= -\csc^2 x
\end{aligned}
$$

$$
\boxed{\frac{d}{dx}\cot x = -\csc^2 x}
$$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\ \frac{1}{\cos x} = -\frac{(\cos x)'}{\cos^2 x} = -\frac{-\sin x}{\cos^2 x}$$

$$= \sec x \tan x$$

$$\boxed{\frac{d}{dx}\sec x = \sec x \tan x}$$

$$\frac{d}{dx}\csc x = \frac{d}{dx}\ \frac{1}{\sin x} = -\frac{(\sin x)'}{\sin^2 x} = -\frac{\cos x}{\sin^2 x}$$

$$= -\csc x \cot x$$

$$\boxed{\frac{d}{dx}\csc x = -\csc x \cot x}$$