Math 303 Winter 2012

Outcome Statements

Homework Assignments

Supplemental Problems

Integrals Expected Known

Graphs for Selected Homework Problems

1 Some Basic Mathematical Models; Direction Fields

Outcomes:

A. Model physical processes using differential equations.
B. Sketch the direction field (or slope field) of a differential equation using a computer graphing program.
C. Describe the behavior of the solutions of a differential equation by analyzing its slope field. Identify any equilibrium solutions.

Reading: Section 1.1

Homework: 1.1: 1,5,7,9,21,22,23; use the direction fields in this handout for #1,5

Outcome Mapping:

A. 21,22,23
B. 1,5
C. 1,5,7,9

2 Solutions of Some Differential Equations; Classification of Differential Equations

Outcomes:

A. Solve basic initial value problems; obtain explicit solutions if possible.
B. Characterize the solutions of a differential equation with respect to initial values.
C. Use the solution of an initial value problem to answer questions about a physical system.
D. Determine the order of an ordinary differential equation. Classify an ordinary differential equation as linear or nonlinear.
E. Verify solutions to ordinary differential equations.
F. Determine the order of a partial differential equation. Classify a partial differential equation as linear or nonlinear.
G. Verify solutions to partial differential equations.

Reading: Sections 1.2 and 1.3

Homework: 1.2: 1,2,9,11(a)(b)(d),15; 1.3: 1-6,7,14,21,22,25,26; use the phase portraits in this handout for #1,2, and the graph in this handout for #11(d)

Outcome Mapping:

A. 1.2: 1,2,11(a)(b),15; in 11(b), be sure to obtain the explicit solution
B. 1.2: 1,2
C. 1.2: 9,11(d),15
D. 1.3: 1-6
E. 1.3: 7,14
F. 1.3: 21,22
G. 1.3: 25,26

3 Linear First Order Equations with Variable Coefficients

Outcomes:

A. Identify and solve first order linear equations.
B. Analyze the behavior of solutions.
C. Solve initial value problems for first order linear equations.

Reading: Section 2.1

Homework: 2.1: 8,11,13,16,25,32; use the direction fields in this handout for #8,11,25

Outcome Mapping:

A. 8,11,25,32
B. 8,11,13,16,25,32
C. 13,16,25,32

4 Separable First Order Equations

Outcomes:

A. Identify and solve separable equations; obtain explicit solutions if possible.
B. Solve initial value problems for separable equations, and analyze their solutions.
C. Apply the transformation $y = xv(x)$ to obtain a separable equation, if possible.

Reading: Section 2.2

Homework: 2.2: 4,7,11,14,21,26,30,32; S1,S2; use the phase portraits in this handout for #30,32,S2

Supplemental Problems:

S1. Can explicit solutions be found for problems #4 and/or #7? If so, find them. If not, why?

S2. The magnetic field lines in the plane containing a magnetic dipole are modeled by

$$\frac{dy}{dx} = \frac{2xy}{3x^2 - y^2}.$$

Solve this equation. [Hint: to solve, use $y = xv(x)$.]

Outcome Mapping:

A. 4,7; S1
B. 11,14,21,26
C. 30,32; S2

5 Modeling with First Order Equations

Outcomes:

A. Construct models of tank problems using differential equations. Analyze the models to answer questions about the physical system modeled.
B. Construct growth and decay problems using differential equations. Analyze the models to answer questions about the physical system modeled.
C. Construct models of problems involving force and motion using differential equations. Analyze the models to answer questions about the physical system modeled.

Reading: Section 2.3

Homework: 2.3: 3,4,18(a)(b),20,21,22(c); in question #21, the quantity $|v|/30$ described as the "force due to air resistance" is actually the magnitude of the force due to air resistance – the actual force due to air resistance is $-v/30$; use the graphs in this handout for #18(b),20,21,22

Outcome Mapping:

A. 3,4
B. 18(a)(b)
C. 20,21,22(c)

6 Differences Between Linear and Nonlinear Equations

Outcomes:

A. Recall and apply the existence and uniqueness theorem for first order linear differential equations.
B. Recall and apply the existence and uniqueness theorem for first order differential equations (both linear and nonlinear).
C. Summarize the nice properties of linear equations. Contrast with nonlinear equations.

Reading: Section 2.4

Homework: 2.4: 2,4,8,14,17,19,22(a)(b),23; S3; use the phase portraits in this handout for #17,19

Supplemental Problem:

S3. Study and then summarize the nice properties of linear equations. Contrast with nonlinear equations.

Outcome Mapping:

A. 2,4
B. 8,14
C. 17,19,22(a)(b),23; S3

7 Autonomous Equations and Population Dynamics (2 days)

Outcomes:

A. Determine and classify the equilibrium solutions of an autonomous equation as asymptotically stable, semistable or unstable by analyzing a graph of $\frac{dy}{dt}$ verses y. Sketch the phase line.
B. Analyze solutions of the logistic equation and other autonomous equations.

Reading: Section 2.5

Homework: 2.5: 3,7,9,14,16(a)(b),17(a),22,23,24,28; S4; in question 17(a), the "change of variable" $u = \ln(y/K)$ is suggested

Supplemental Problem:

S4. [Reiss (1980), Kassoy, Kapila] A model for the combustion of a chemical reactant is

$$y' = y^2(1-y), \quad 0 \le y \le 1,$$

where y is the concentration of the reactant, $y = 0$ is the preignited state, and $y = 1$ is the ignited state. Sketch the phase line for this equation, and find the value(s) of y for which the solutions change concavity. Then solve the equation subject to the initial condition $y(0) = 0.0001$. [The implicit solution you should get makes a rapid rise from near $y = 0$ to near $y = 1$ at time $t = 10000$ – the chemical reactant ignites "suddenly" after a "long" time.]

Outcome Mapping:

A. 3,7,9,14,16(a)(b),17(a); S4
B. 16(a)(b),17(a),22,23,24,28; S4

8 Exact Equations and Integrating Factors

Outcomes:

A. Identify whether or not a differential equation is exact.
B. Solve exact differential equations with or without initial conditions, and obtain explicit solutions if possible.
C. Use integrating factors to convert a differential equation to an exact equation and then solve.
D. Determine an integrating factor of the form $\mu(x)$ or $\mu(y)$ which will convert a non-exact differential equation to an exact equation, if possible.

Reading: Section 2.6

Homework: 2.6: 1,4,11,15,19,23,25,30; S5

Supplemental Problem:

S5. Find the explicit solutions of the differential equations in questions #1 and #4 subject to the initial conditions (a) $y(1) = 2$, (b) $y(1) = -2$. [Hint: use the quadratic formula, and choose the sign correctly.]

Outcome Mapping:

A. 1,4,11,15
B. 1,4,11,15; S5
C. 19
D. 23,25,30

9 Introduction to Second Order Equations

Outcomes:

A. Determine the characteristic equation of a second order linear differential equation with constant coefficients.
B. Solve second order linear differential equations with constant coefficients that have a characteristic equation with real and distinct roots.
C. Describe the behavior of solutions.
D. Convert a second order differential equation to a first order differential equation in the following cases:

   i) $y'' = f(t, y')$,
   ii) $y'' = f(y, y')$.

Reading: Section 3.1

Homework: 3.1: 2,8,10,16; 2.9 Misc. Problems (on p.134-135): 37,44,48,51; S6

Supplemental Problem:

S6. In modeling the shape of a cable suspended between two poles (not necessarily of the same height), one obtains

$$y'' = \frac{1}{a}\sqrt{1+(y')^2}, \quad y(0) = a, \quad y'(0) = 0,$$

where a is a nonzero constant. Solve this initial value problem. [The graph of the solution is called a catenary. The answer is $y = a\cosh(t/a)$. Hint: t is missing in the ODE. Use $v = y'$ and $v' = y''$ to get a separable first order ODE, and then use $v = \sinh(u)$ to integrate.]

Outcome Mapping:

A. 3.1: 2,8,10,16
B. 3.1: 2,8,10,16
C. 3.1: 10,16
D. 2.9 (p.133): 37,44,48,51; S6

10 Fundamental Solutions of Linear Homogeneous Equations; the Wronskian (2 days)

Outcomes:

A. Recall and apply the existence and uniqueness theorem for second order linear differential equations.
B. Recall and verify the principal of superposition for solutions of second order linear differential equations.
C. Evaluate the Wronskian of two functions.
D. Determine whether or not a pair of solutions of a second order linear differential equations constitute a fundamental set of solutions.
E. Recall and apply Abel's theorem.

Reading: Section 3.2

Homework: 3.2: 3,6,10,12,13,14,24,25,31,32,33,34,35,36

Outcome Mapping:

A. 10,12
B. 13,14
C. 3,6
D. 24,25
E. 31,32,33,34,35,36

11 Complex Roots of the Characteristic Equation

Outcomes:

A. Use Euler's formula to rewrite complex expressions in different forms.
B. Solve second order linear differential equations with constant coefficients that have a characteristic equation with complex roots.
C. Solve initial value problems and analyze the solutions.

Reading: Section 3.3

Homework: 3.3: 2,6,11,12,17,18,19,24(a)

Outcome Mapping:

A. 2,6
B. 11,12,17,18,19,24(a)
C. 17,18,19

12 Repeated Roots; Reduction of Order

Outcomes:

A. Solve second order linear differential equations with constant coefficients that have a characteristic equation with repeated roots.
B. Solve initial value problems and analyze the solutions.
C. Apply the method of reduction of order to find a second solution to a given differential equation.

Reading: Section 3.4

Homework: 3.4: 1,4,8,11,16,17,27,28

Outcome Mapping:

A. 1,4,8
B. 11,16,17
C. 27,28

13 Nonhomogeneous Equations; Method of Undetermined Coefficients

Outcomes:

A. For a nonhomogeneous second order linear differential equation, determine a suitable form of a particular solution that can be used in the method of undetermined coefficients.
B. Apply the method of undetermined coefficients to solve nonhomogeneous second order linear differential equations.
C. Solve initial value problems and analyze the solutions.

Reading: Section 3.5

Homework: 3.5: 2,3,6,13,17,19(a),23(a); S7

Supplemental Problem:

S7. For questions #3 and #6, what are the possibilities for the limits of the solutions as $t \to \infty$? How do the limits depend on the choice of the arbitrary constants?

Outcome Mapping:

A. 2,3,6,13,17,19(a),23(a)
B. 2,3,6,13,17,19(a),23(a)
C. 13,17; S7

14 Variation of Parameters; Reduction of Order

Outcomes:

A. Apply the method of variation of parameters to solve nonhomogeneous second order linear differential equations with or without initial conditions.
B. Apply the method of reduction of order to solve nonhomogeneous second order linear differential equations with or without initial conditions.

Reading: Section 3.6

Homework: 3.6: 5,8,15,17,28; S8,S9

Supplemental Problems:

S8. Find the solution of nonhomogeneous equation in question #15 that satisfies $y(1) = 1$, $y'(1) = 1$.

S9. Use reduction of order to find the general solution of

$$ty'' - (2t+1)y' + 2y = 8t^2e^{2t}, \quad t > 0,$$

given that $y_1(t) = e^{2t}$ is a solution of associated homogeneous equation. Then find the solution of nonhomogeneous equation that satisfies $y(1) = 1$, $y'(1) = 1$.

Outcome Mapping:

A. 5,8,15,17; S8
B. 28; S9

15 Mechanical Vibrations (2 days)

Outcomes:

A. Model undamped mechanical vibrations with second order linear differential equations, and then solve. Analyze the solution. In particular, evaluate the frequency, period, amplitude, phase shift, and the position at a given time.
B. Model damped mechanical vibrations with second order linear differential equations, and then solve. Analyze the solution. In particular, evaluate the quasi frequency, quasi period, and the behavior at infinity.
C. Define critically damped and overdamped. Identify when these conditions exist in a system.

Reading: Section 3.7

Homework: 3.7: 5,6,7,11,13,17,20,24; S10 Do not plot in #5

Supplemental Problem:

S10. A damped mass-spring system can be used to model the suspension of a car, in which the mass of the car is supported by a shock absorber and a spring. If the mass of the car is $m = 1000$ kg and the spring constant is $k = 3000$ kg/sec^2, determine the minimum value of the damping constant γ of the shock absorber that will provide an oscillation-free suspension. If the spring is replaced by a heavy duty one having $k = 6000$ kg/sec^2, how does the minimum value for γ change?

Outcome Mapping:

A. 5,6,7,24
B. 11,13
C. 17,20; S10

16 Forced Vibrations (2 days)

Outcomes:

A. Model forced, undamped mechanical vibrations with second order linear differential equations, and then solve. Analyze the solution.
B. Describe the phenomena of beats and resonance. Determine the frequency at which resonance occurs.

C. Model forced, damped mechanical vibrations with second order linear differential equations, and then solve. Determine and analyze the solutions, including the steady state and transient parts.

Reading: Section 3.8

Homework: 3.8: 2,5,6,7(a)(c),8(a)(b)(d),9,10,11,12; S11,S12

Supplemental Problems:

S11. During and earthquake, the horizontal displacement in feet, $u(t)$, of the second floor of a two story building is modeled by

$$mu'' + ku = mA_0\omega^2 \sin\omega t, \quad u(0) = 0, \quad u'(0) = 0,$$

where m is the mass of the second floor, k is the spring constant for the elastic frame of the building, and A_0 is the amplitude and ω is the frequency of the horizontally oscillating ground. Suppose $m = 1000$ slugs and $k = 20000$ slugs/sec^2. What is the natural frequency of the second floor? If $A_0 = 0.25$ feet and $\omega = 3$ Hz, find $u(t)$ for $t > 0$. Plot this solution and determine from its graph what the maximum horizontal displacement of the second floor is. [Interpret ωt in radians. Answer: approximately 0.34 feet.]

S12. The torsional motion of an object suspended from the end of an elastic shaft is modeled by

$$I\theta'' + c\theta' + k\theta = T(t), \quad \theta(0) = \theta_0, \quad \theta'(0) = \theta_0'$$

where $\theta(t)$ is the amount of twist, or angle, of the object at time t, I is the moment of inertia of the object, c is the damping constant, k is the elastic shaft constant, and $T(t)$ is the applied torque. If $I = 1$, $c = 4$, $k = 13$, and $T(t) = \sin 3t$, find $\theta(t)$ for $t > 0$ when $\theta_0 = 0$ and $\theta_0' = 0$.

Outcome Mapping:

A. 5,7(a)(c),9,10; S11
B. 2,7(a)(c)
C. 6,8(a)(b)(d),11,12; S12

17 General Theory of n^{th} Order Linear Equations

Outcomes:

A. Recall and apply the existence and uniqueness theorem for higher order linear differential equations.
B. Recall the definition of linear independence for a finite set of functions. Determine whether a set of functions is linearly independent or linearly dependent.
C. Use the Wronskian to determine if a set of solutions form a fundamental set of solutions.
D. Recall the relationship between Wronskian and linear independence for a set of functions, and for a set of solutions.
E. Apply the method of reduction of order to solve higher order linear differential equations.

Reading: Section 4.1

Homework: 4.1: 3,8,14,15,17,25,28; S13

Supplementary Problem:

S13. In questions 14 and 15, do the given functions form a fundamental set of solutions for the given equations?

Outcome Mapping:

A. 3
B. 8,25
C. 14,15,25; S13
D. 8,14,15,17
E. 28

18 Homogeneous Equations with Constant Coefficients

Outcomes:

A. Apply Euler's formula to write a complex number in exponential form. Find the distinct complex roots of a number.
B. Determine the characteristic equation of higher order linear differential equations.
C. Solve higher order linear differential equations.
D. Solve initial value problems.

Reading: Section 4.2

Homework: 4.2: 1,2,8,9,11,17,24,29,31 Do not plot in #29,31

Outcome Mapping:

A. 1,2,8,9
B. 11,17,24
C. 11,17,24
D. 29,31

19 The Method of Undetermined Coefficients

Outcomes:

A. For a nonhomogeneous higher order linear differential equation, determine a suitable form of a generalized particular solution that can be applied in the method of undetermined coefficients.
B. Use the method of undetermined coefficients to solve nonhomogeneous higher order linear differential equations.
C. Solve initial value problems.

Reading: Section 4.3

Homework: 4.3: 2,5,11,14,17; S14 Do not plot in #11

Supplemental Problem:

S14. The deflection of a beam under a load and subject to a constant axial force is modeled by

$$y^{(4)} - k^2 y'' = g(x), \quad 0 < x < L,$$

where $y(x)$ is the deflection of the beam, L is the length of the beam, $k^2 > 0$ is proportional to the axial force, and $g(x)$ is proportional to the load. Find the general solution if $g(x) = 1$.

Outcome Mapping:

A. 2,5,11,14,17
B. 2,5,11; S14
C. 11

20 The Method of Variation of Parameters

Outcomes:

A. Use the method of variation of parameters to solve nonhomogeneous higher order linear differential equations.
B. Solve initial value problems.

Reading: Section 4.4

Homework: 4.4: 1,7,9,12,13; S15 Do not plot in #9,12

Supplemental Problem:

S15. The deflection of a beam under a load and subject to a constant axial force is modeled by

$$y^{(4)} - k^2 y'' = g(x), \quad 0 < x < L,$$

where $y(x)$ is the deflection of the beam, L is the length of the beam, $k^2 > 0$ is proportional to the axial force, and $g(x)$ is proportional to the load. Find the general solution when $g(x)$ is arbitrary.

Outcome Mapping:

A. 1,7,12,13; S15
B. 9,12

21 Review of Power Series

Outcomes:

A. Determine the radius of convergence of a power series.
B. Find the power series expansion of a function.
C. Manipulate expressions involving summation notation. Change the index of summation.

Reading: Section 5.1

Homework: 5.1: 5,8,12,15,17,18,19,25,27,28

Outcome Mapping:

A. 5,8
B. 12,15
C. 17,18,19,25,27,28

22 Series Solutions near an Ordinary Point, Part I

Outcomes:

A. Find the general solution of a differential equation using power series.
B. Solve initial value problems. Analyze the solution.

Reading: Section 5.2

Homework: 5.2: 2(a)(b)(c),6(a)(b)(c),15(a),16(a)

Outcome Mapping:

A. 2(a)(b),6(a)(b)
B. 2(c),6(c),15(a),16(a)

23 Series Solutions near an Ordinary Point, Part II

Outcomes:

A. Given an initial value problem, use the differential equation to inductively determine the terms in the power series of the solution, expanded about the initial value.
B. Determine a lower bound for the radius of convergence of a series solution.

Reading: Section 5.3

Homework: 5.3: 1,3,7,11,12

Outcome Mapping:

A. 1,3,11,12
B. 7,11,12

24 Euler Equations

Outcomes:

A. Find the general solution to an Euler equation in the cases of

   i. real distinct roots

   ii. equal roots

   iii. complex roots

B. Solve initial value problems for Euler equations and analyze their solutions.

Reading: Section 5.4

Homework: 5.4: 1,3,10,13,14,15 Do not plot in #13,14,15

Outcome Mapping:

A. 5.5: 1,3,10
B. 5.5: 13,14,15

25 Definition of Laplace Transform

Outcomes:

A. Sketch a piecewise defined function. Determine if it is continuous, piecewise continuous or neither.
B. Evaluate Laplace transforms from the definition.
C. Determine whether an infinite integral converges or diverges.

Reading: Section 6.1

Homework: 6.1: 1,2,3,5(a)(b),6,7,10,22,23

Outcome Mapping:

A. 1,2,3
B. 5(a)(b),6,7,10
C. 22,23

26 Solution of Initial Value Problems

Outcomes:

A. Evaluate inverse Laplace transforms.
B. Use Laplace transforms to solve initial value problems.
C. Evaluate Laplace transforms using the derivative identity given in Problem 28 (p. 322).

Reading: Section 6.2

Homework: 6.2: 4,5,8,11,16,21,29,30,31

Outcome Mapping:

A. 4,5,8
B. 11,16,21
C. 29,30,31

27 Step Functions

Outcomes:

A. Sketch the graph of a function that is defined in terms of step functions.
B. Convert piecewise defined functions to functions defined in terms of step functions and vice versa.
C. Find the Laplace transform of a piecewise defined function.
D. Apply the shifting theorems (Theorems 6.3.1 and 6.3.2) to evaluate Laplace transforms and inverse Laplace transforms.

Reading: Section 6.3

Homework: 6.3: 3,6,15,17,19,21; S16

Supplemental Problem:

S16. Write the functions in problems 3 and 6 as piecewise defined functions.

Outcome Mapping:

A. 3,6
B. 15,S16
C. 15,17
D. 19,21

28 Differential Equations with Discontinuous Forcing Functions

Outcomes:

A. Use Laplace transforms to solve differential equations with discontinuous forcing functions.
B. Analyze the solutions of differential equations with discontinuous forcing functions.

Reading: Section 6.4

Homework: 6.4: 2(a),3(a),11(a),19(a)(b); S17

Supplemental Problem:

S17. A space module of mass 1250 kg is launched from its mothership directly towards a moon (without atomsphere) on which the space module is to land. The mothership is 11805 meters above the moon's surface, and the space module is launched with a speed of 48 meters per second. The space module is equipped with thrusters which when engaged produce a force of 9500 N. The space module's vertical descent towards the moon's surface is modeled by

$$1250y'' = -4625 + 9500u_c(t),$$
$$y(0) = 11805,$$
$$y'(0) = -48,$$

where y is the position (in meters) of the space module above the moon's surface at time t (in seconds), and where c is the time at which the thrusters are engaged. The descent of the space module is to take 100 seconds (i.e., $y(100) = 0$). Find the time c which will bring the space module to a "soft" landing on the moon (i.e., determine the value of c for which $y'(100) = 0$). [Answer: $c = 45$.]

Outcome Mapping:

A. 2(a),3(a),11(a); S17
B. 19(a)(b); S17

29 Impulse Functions

Outcomes:

A. Define an idealized unit impulse function.
B. Use Laplace transforms to solve differential equations that involve impulse functions.
C. Analyze the solutions of differential equations that involve impulse functions.

Reading: Section 6.5

Homework: 6.5: 2(a),3(a),11(a),13(a),16(a)(b)(c),18(a)(c); S18

Supplemental Problem:

S18. For positive constants t_0 and k, what is the graph and impulse of

$$g_k(t) = u_{t_0-k}(t)\left[\frac{t-(t_0-k)}{k^2}\right] - 2u_{t_0}(t)\left[\frac{t-t_0}{k^2}\right] + u_{t_0+k}(t)\left[\frac{t-(t_0+k)}{k^2}\right]?$$

What is the limit of the $\mathcal{L}\{g_k(t)\}$ as $k \to 0^+$? What "function" does $g_k(t)$ describe as $k \to 0^+$?

Outcome Mapping:

A. 16(a)(b)(c); S18
B. 2(a),3(a),11(a)
C. 13(a),16(a)(b)(c),18(a)(c)

30 The Convolution Integral

Outcomes:

A. Evaluate the convolution of two functions from the definition.
B. Prove and disprove properties of the convolution.
C. Evaluate the Laplace transform of a convolution of functions.
D. Use the convolution theorem to evaluate inverse Laplace transforms.
E. Solve initial value problems using convolution.

Reading: Section 6.6

Homework: 6.6: 1(a)(b),2,3,5,6,9,10,13,15; S19

Supplemental Problem:

S19. Evaluate problems 5 and 6 directly without using Theorem 6.6.1.

Outcome Mapping:

A. S19
B. 1(a)(b),2,3
C. 5,6
D. 9,10
E. 13,15

31 Introduction to Systems of First Order Equations

Outcomes:

A. Transform a higher order linear differential equation into a system of first order linear equations.
B. Transform a system of first order linear equations to a single higher order linear equation.
C. Model physical systems that involve more than one unknown function with a system of differential equations.
D. Recall and apply methods of linear algebra

Reading: Section 7.1-3

Homework: 7.1: 2,4,7(a)(b),8(a)(b),18,22; 7.2: 21,24,25; 7.3: 13,15,16,17,23,25

Outcome Mapping:

A. 7.1: 2,4,18
B. 7.1: 7(a)(b),8(a)(b)
C. 7.1: 22
D. 7.2: 21,24,25; 7.3: 13,15,16,17,23,25

32 Basic Theory of Systems of First Order Linear Equations

Outcomes:

A. Recall and verify the superposition principle for first order linear systems.
B. Relate the Wronskian to linear independence and a fundamental set of solutions.

Reading: Section 7.4

Homework: 7.4: 1,6(a)(b),7(a)(b); S20

Supplemental Problem:

S20. Are $\mathbf{x}^{(1)}(t) = \begin{bmatrix} 1/t \\ 2/t \end{bmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{bmatrix} 2t^2 \\ t^2 \end{bmatrix}$ a fundamental set of solutions for $\mathbf{x}' = \begin{bmatrix} 3/t & -2/t \\ 2/t & -2/t \end{bmatrix} \mathbf{x}$?

Outcome Mapping:

A. 1; S20
B. 6(a)(b),7(a)(b); S20

33 Homogeneous Linear Systems with Constant Coefficients

Outcomes:

A. Sketch a direction field and a phase portrait for a homogeneous linear system with constant coefficients.
B. Find the general solution of a homogeneous linear system with constant coefficients in the case of real, distinct eigenvalues.
C. Determine if the origin is a saddle point or a node for a homogeneous linear system. Classify a node as asymptotically stable or unstable.
D. Find general solutions, solve initial value problems, and analyze their solutions.

Reading: Sections 7.5, 9.1

Homework: 7.5: 1,2,7,16,18,30(a); 9.1: 2(a,b,c),3(a,b,c),8(a,b,c) Do not sketch the graph of x_1 versus t for 9.1 #2(c),3(c),8(c)

Outcome Mapping:

A. 7.5: 1,2,7; 9.1: 2(a,b,c),3(a,b,c),8(a,b,c)
B. 7.5: 1,2,7
C. 9.1: 2(a,b,c),3(a,b,c),8(a,b,c)
D. 7.5: 16,18,30(a)

34 Complex Eigenvalues

Outcomes:

A. Sketch a direction field and a phase portrait for a homogeneous linear system with constant coefficients.
B. Find the general solution of a homogeneous linear system with constant coefficients in the case of complex eigenvalues.
C. Classify the origin as a saddle point, a node, a spiral point or a center.
D. Solve and analyze physical problems modeled by systems of differential equations.

Reading: Sections 7.6, 9.1

Homework: 7.6: 1,2,7,13; 9.1: 5(a,b,c),6(a,b,c),7(a,b,c); S21 Do not sketch the graph of x_1 versus t in 9.1 #5(c),6(c),7(c)

Supplementary Problem:

S21. A diatomic molecule such as O_2 in which both atoms are constrained to move along a line is modeled by the pair of second order nonhomogeneous equations

$$mx_1'' = k(x_2 - x_1 - d), \quad mx_2'' = k(x_1 - x_2 + d),$$

where m is the mass of either atom, x_1 is the position of the first atom, x_2 is the position of the second atom, d is the equilibrium bond length, and k is the bond constant. Convert this pair of second order equations into a system of four first order linear homogeneous equations by setting $y_1 = x_1$, $y_2 = x_2 - d$, $y_3 = x_1'$, and $y_4 = x_2'$. Find the eigenvalues for the resulting 4×4 matrix. [Answer: the eigenvalues are 0,0, $\pm i\sqrt{2k/m}$.] Find the solutions corresponding to the eigenvalues $\pm i\sqrt{2k/m}$. [These solutions are the normal modes of vibration for the diatomic molecule.]

Outcome Mapping:

A. 7.6: 1,2
B. 7.6: 1,2,7
C. 7.6: 13; 9.1: 5(a,b,c),6(a,b,c),7(a,b,c)
D. 7.6: S21

35 Fundamental Matrices

Outcomes:

A. Determine a fundamental matrix for a system of equations.
B. Solve initial value problems using a fundamental matrix.
C. Prove identities using fundamental matrices.

Reading: Section 7.7

Homework: 7.7: 3,5,6,11,12,14,15

Outcome Mapping:

A. 3,5,6
B. 11,12
C. 14,15

36 Repeated Eigenvalues

Outcomes:

A. Sketch a direction field and a phase portrait for a homogeneous linear system with constant coefficients.
B. Find the general solution of a homogeneous linear system with constant coefficients in the case of repeated eigenvalues.
C. Identify improper nodes. Classify them as asymptotically stable or unstable.
D. Solve initial value problems.

Reading: Sections 7.8, 9.1

Homework: 7.8: 1(c),4(c),5(a),8(a),11(a); 9.1: 4(a,b,c),9(a,b,c),11(a,b,c) Do not sketch the graph of x_1 versus t in 9.1 #4(c),9(c),11(c)

Outcome Mapping:

A. 7.8: 1(c),4(c)
B. 7.8: 1(c),4(c),5(a)
C. 9.1: 4(a,b,c),9(a,b,c),11(a,b,c)
D. 7.8: 8(a),11(a)

37 Nonhomogeneous Linear Systems

Outcomes:

A. Use diagonalization to solve nonhomogeneous linear systems.
B. Use the method of undetermined coefficients to solve nonhomogeneous linear systems.
C. Use the method of variation of parameters to solve nonhomogeneous linear systems.
D. Solve initial value problems.

Reading: Section 7.9

Homework: 7.9: 1,8,11,12,14; S22

Supplementary Problem:

S22. For questions #1 and #11, find the solution of the system that satisfies $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Outcome Mapping:

A. 1,8
B. 11
C. 11,12,14
D. S22

38 Two-Point Boundary Value Problems

Outcomes:

A. Solve boundary value problems involving linear differential equations.
B. Find the eigenvalues and the corresponding eigenfunctions of a boundary value problem.

Reading: Section 10.1

Homework: 10.1: 3,4,6,10,11,14,15,18; S23

Supplementary Problem:

S23. Find the eigenvalues and associated eigenfunctions for the BVP

$$\begin{aligned} &y'' + \lambda y = 0, \\ &y(0) = 0, \\ &y(1) + y'(1) = 0. \end{aligned}$$

[Hint: if $\lambda = \mu^2 > 0$, then μ is implicitly given by $f(\mu) = 0$ for a function f. Find f.]

Outcome Mapping:

A. 3,4,6,10,11
B. 14,15,18; S23

39 Fourier Series (2 days)

Outcomes:

A. Identify functions that are periodic. Determine their periods.
B. Find the Fourier series for a function defined on a closed interval.
C. Determine the mth partial sum of the Fourier series of a function. Compare to the function.

Reading: Section 10.2

Homework: 10.2: 1,3,4,7,8,13,14,15,18,19,20,22; use graphs in this handout for #19,20,22

Outcome Mapping:

A. 1,3,4,7,8
B. 13,14,15,18,19,20,22
C. 19,20,22

40 The Fourier Convergence Theorem

Outcomes:

A. Find the Fourier series for a periodic function.
B. Recall and apply the convergence theorem for Fourier series.

Reading: Section 10.3

Homework: 10.3: 2,3,5,7,9,10; use graphs in this handout for #7,9,10

Outcome Mapping:

A. 2,3,5,7,9,10
B. 2,3,5,7,9,10

41 Even and Odd Functions

Outcomes:

A. Determine whether a given function is even, odd or neither.
B. Sketch the even and odd extensions of a function defined on the interval $[0, L]$.
C. Find the Fourier sine and cosine series for the function defined on $[0, L]$.
D. Establish identities involving infinite sums from Fourier series.

Reading: Section 10.4

Homework: 10.3: 17; 10.4: 1,3,5,7,8,11,15,16,17,18,35,36

Outcome Mapping:

A. 10.4: 1,3,5
B. 10.4: 7,8,11
C. 10.4: 15,16,17,18
D. 10.3: 17; 10.4: 35,36

42 Separation of Variables; Heat Conduction in a Rod

Outcomes:

A. Apply the method of separation of variables to solve partial differential equations, if possible.
B. Find the solutions of heat conduction problems in a rod using separation of variables.

Reading: Section 10.5

Homework: 10.5: 1,3,5,7,9,10,22

Outcome Mapping:

A. 1,3,5,22
B. 7,9,10

43 Other Heat Conduction Problems

Outcomes:

A. Solve steady state heat conduction problems in a rod with various boundary conditions.
B. Analyze the solutions.

Reading: Section 10.6

Homework: 10.6: 1,3,6,7,9(a),12(a)(b),14(a)

Outcome Mapping:

A. 1,3,6,7,12
B. 9(a),12(a)(b),14(a)

44 The Wave Equation; Vibrations of an Elastic String

Outcomes:

A. Solve the wave equation that models the vibration of a string with fixed ends.
B. Describe the motion of a vibrating string.

Reading: Section 10.7

Homework: 10.7: 1(a),2(a),5(a),6(a),15(b)(c)

Outcome Mapping:

A. 1(a),2(a),5(a),6(a)
B. 15(b)(c)

45 Laplace's Equation (2 days)

Outcomes:

A. Solve Laplace's equation over a rectangular region for various boundary conditions.
B. (Exam Exempt) Solve Laplace's equation over a circular region for various boundary conditions.

Reading: Section 10.8

Homework: 10.8: 1(a)(b),3(a),10,11; S24

Supplementary Problem:

S24. Solve Laplace's equation $u_{rr} + (1/r)u_r + (1/r^2)u_{\theta\theta} = 0$ subject to boundary condition $u(a, \theta) = \theta$, $0 < \theta < 2\pi$ and a constant $a > 0$.

Outcome Mapping:

A. 1(a)(b),3(a),10
B. (Exam Exempt) 11; S24

Integrals Expected Known

Throughout the course, you will be integrating many functions. You will be using partial fraction decompositions and integration by parts for many of these integrations. In addition to the integrals of elementary functions (such as power functions, polynomials, exponentials, sines, and cosines – which integrals you should know by now), you are expected to known and use on homework and examinations the following integrals:

$$\int \sec x \; dx = \ln|\sec x + \tan x| + C,$$

$$\int \csc x \; dx = \ln|\csc x - \cot x| + C = -\ln|\csc x + \cot x| + C,$$

$$\int \tan x \; dx = -\ln|\cos x| + C,$$

$$\int \cot x \; dx = \ln|\sin x| + C,$$

$$\int \ln x \; dx = x \ln x - x + C,$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C,$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C,$$

$$\int e^{ax} \sin bx \; dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C,$$

$$\int e^{ax} \cos bx \; dx = \frac{e^{ax}(b \sin bx + a \cos bx)}{a^2 + b^2} + C,$$

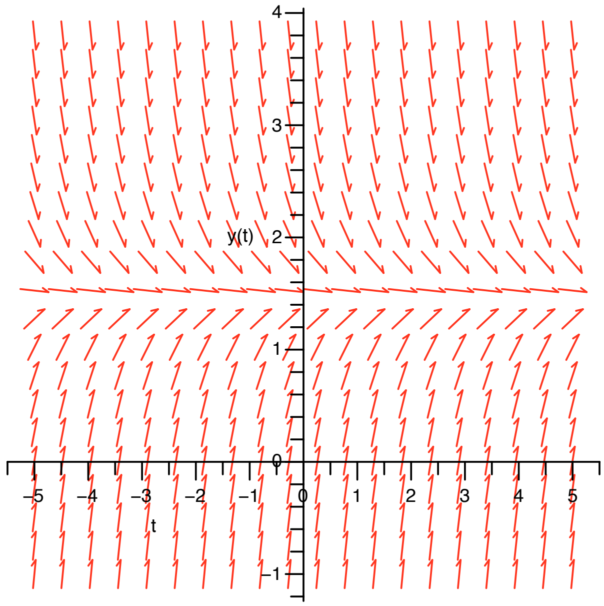
$$\int \sin^2 mx \; dx = \frac{mx - \sin mx \cos mx}{2m} + C,$$

$$\int \cos^2 mx \; dx = \frac{mx + \sin mx \cos mx}{2m} + C.$$

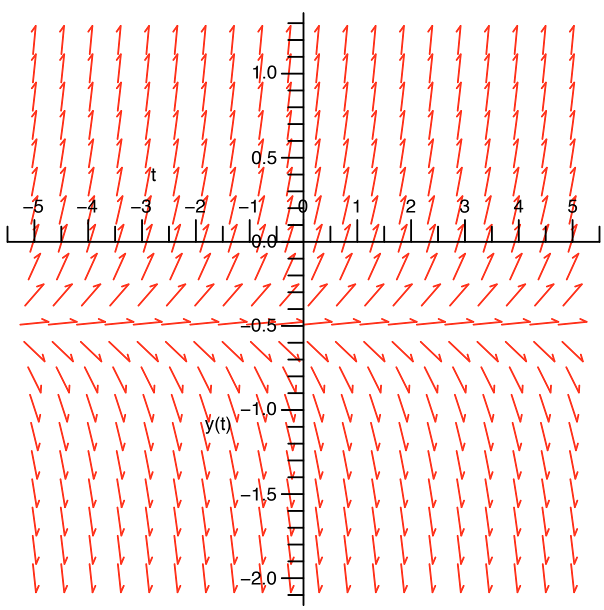
Graphs for Selected Homework Problems

Direction Fields for Homework §1.1

#1. A direction field for $y' = 3 - 2y$ is

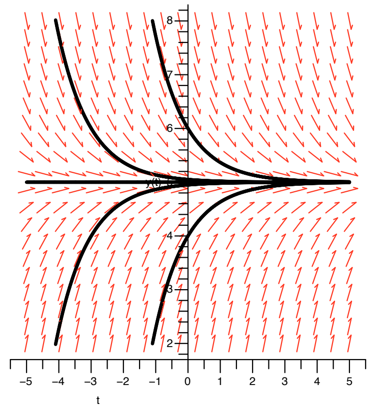


#5. A direction field for $y' = 1 + 2y$ is

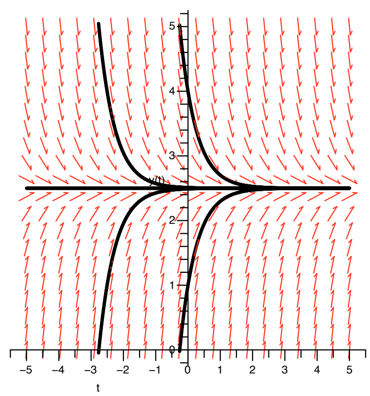


Graphs for Homework §1.2,1.3

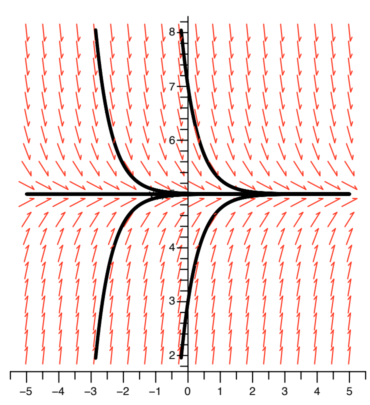
#1(a). A direction field for $y' = -y + 5$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



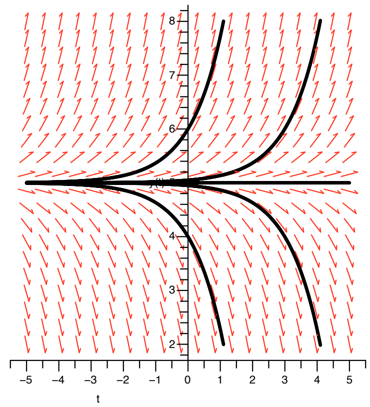
#1(b). A direction field for $y' = -2y + 5$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



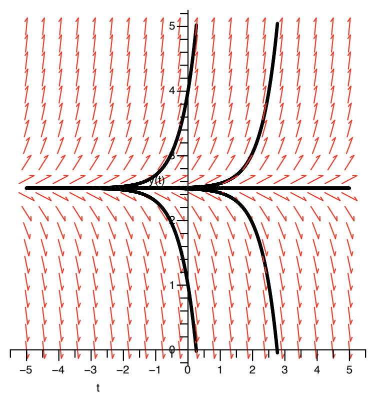
#1(c). A direction field for $y' = -2y + 10$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



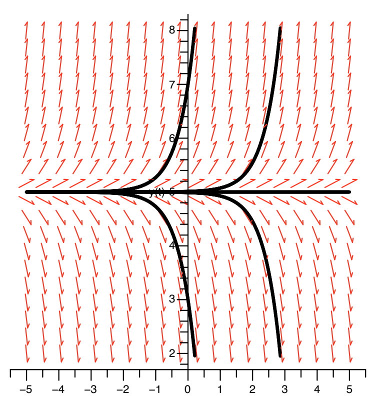
#2(a). A direction field for $y' = y - 5$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



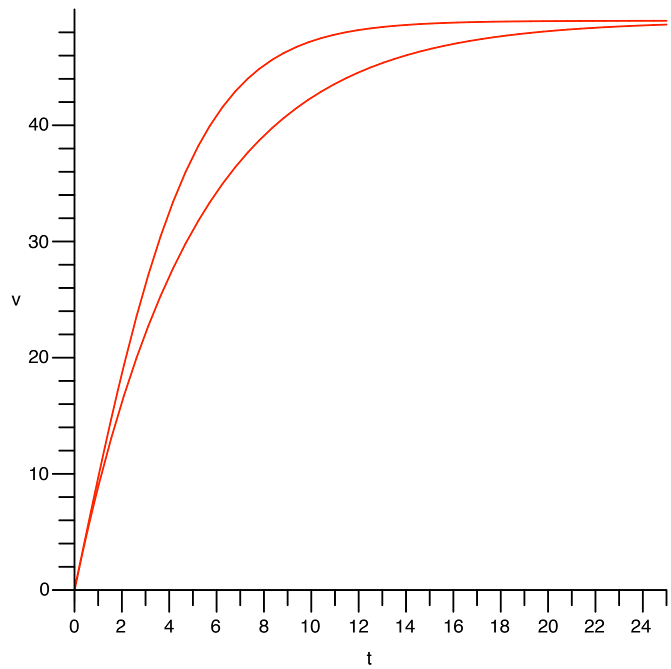
#2(b). A direction field for $y' = 2y - 5$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



#2(c). A direction field for $y' = 2y - 10$ along with some solutions satisfying the initial condition $y(0) = y_0$ for a variety of values of y_0.



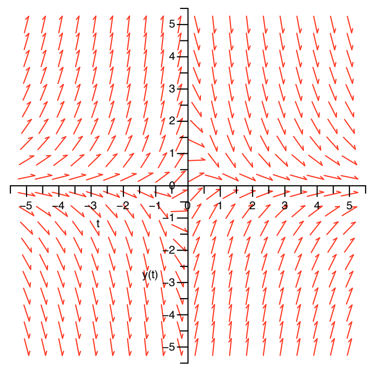
#11(c). Plots of the solution (26) in the book on p. 14, and your answer from #11(b).



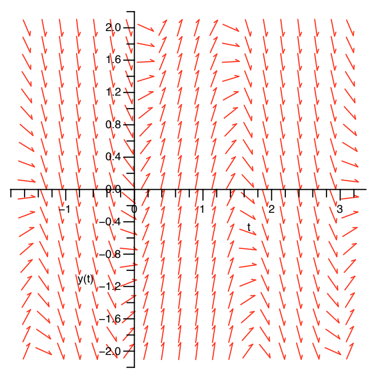
Which one is which?

Direction Fields for Homework §2.1

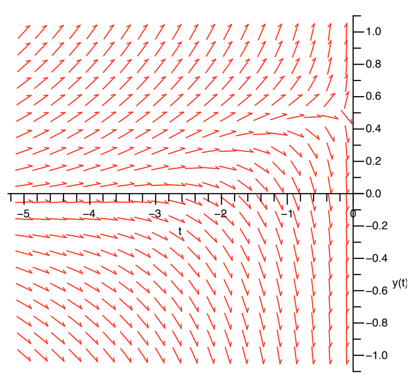
#8(a). A direction field for $(1+t^2)y' + 4ty = (1+t^2)^{-2}$.



#11(a). A direction field for $y' + y = 5\sin 2t$.

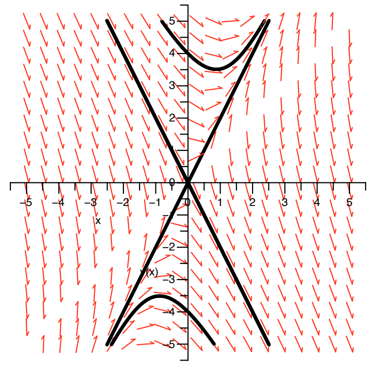


#25(a). A direction field for $ty' + 2y = (\sin t)/t$.

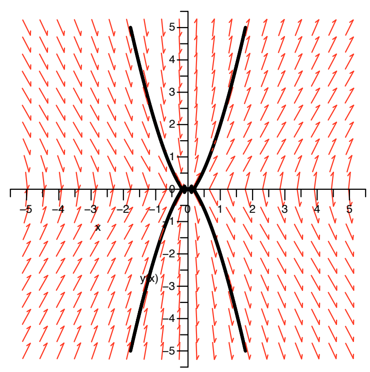


Direction Fields for Homework §2.2

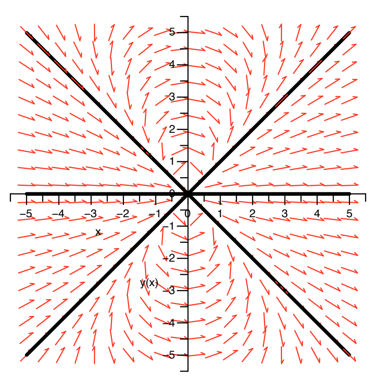
#30(f). A direction field for $\dfrac{dy}{dx} = \dfrac{y-4x}{x-y}$.



#32(c). A direction field for $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy}$.

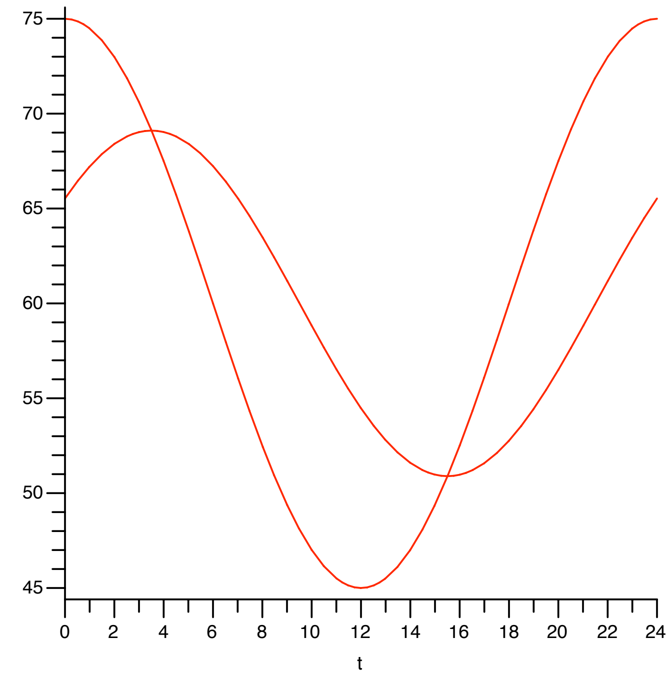


#**S2**. A direction field for $\dfrac{dy}{dx} = \dfrac{2xy}{3x^2-y^2}$.

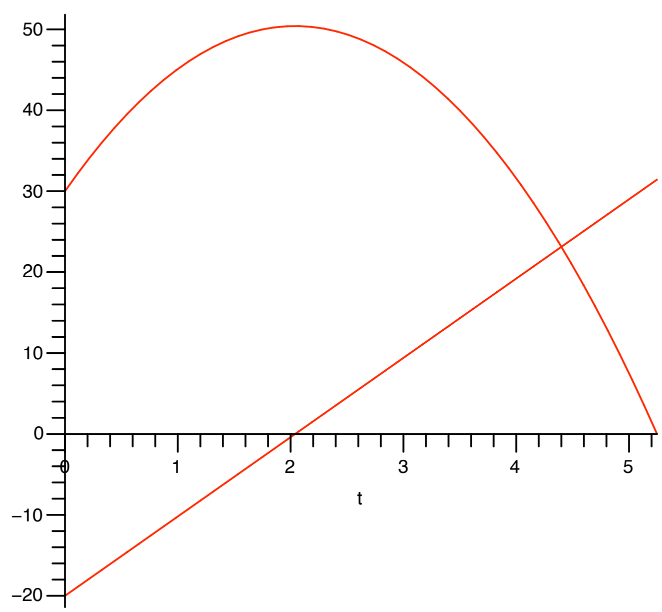


Graphs for Homework §2.3

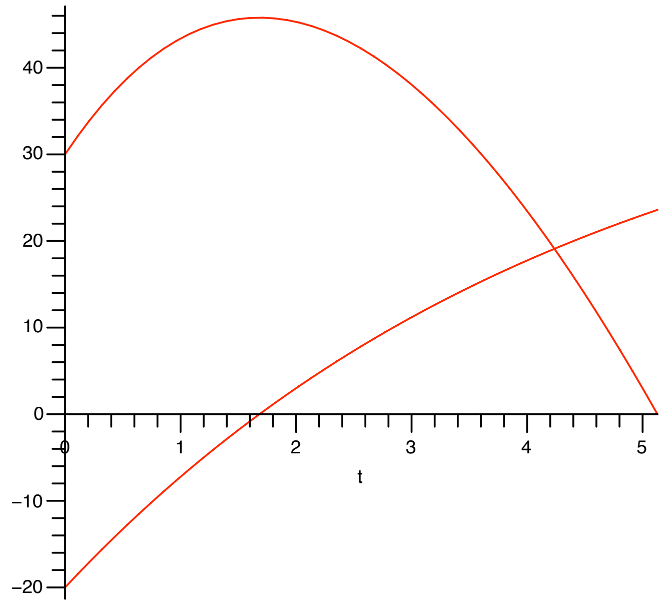
#18(b). The graphs of $T(t)$ and $S(t)$ on the same axes.



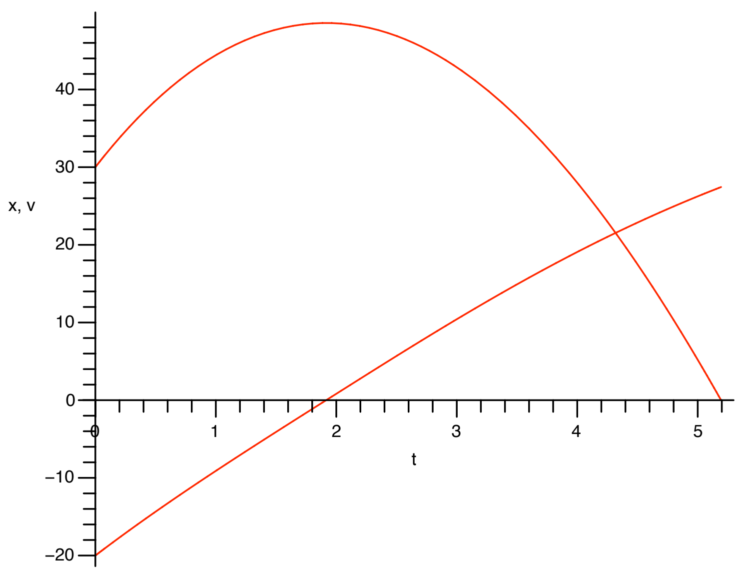
#20(c). The graphs of velocity and position on the same axes. The position is plotted as distance above ground.



#21(c). The graphs of velocity and position on the same axes. The position is plotted as distance above ground.

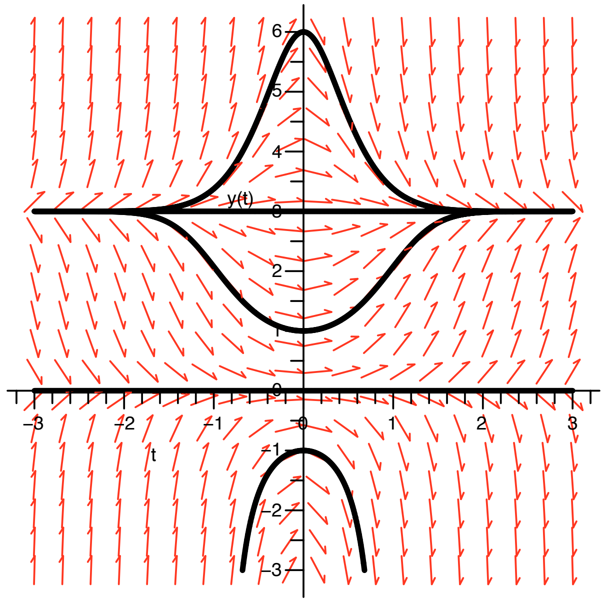


#22 (c). The graphs of velocity and position on the same axes. The position is plotted as distance above ground.

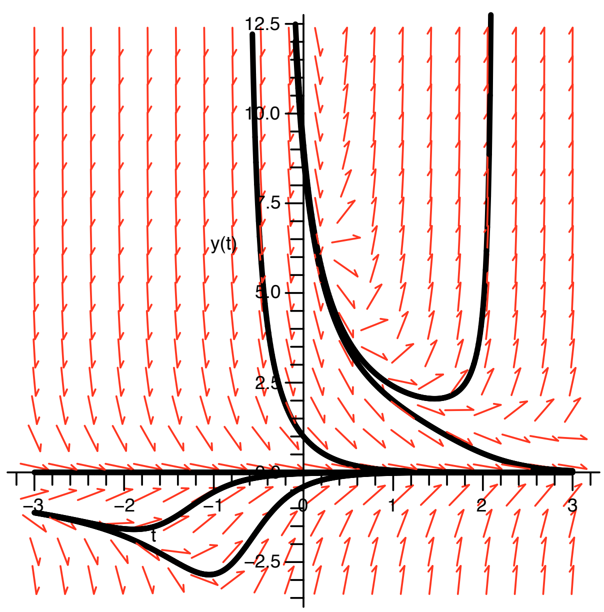


Phase Portraits for Homework §2.4

#17. A phase portrait for $y' = ty(3 - y)$.

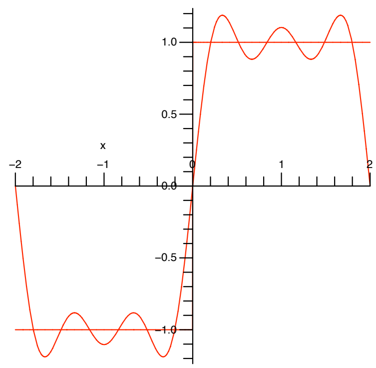


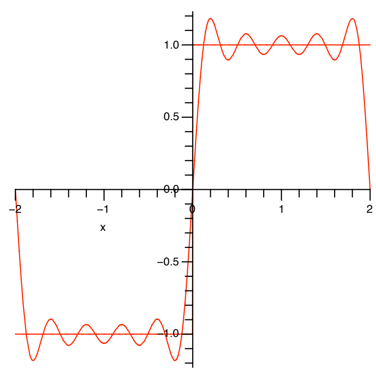
#19. A phase portrait for $y' = -y(3 - ty)$.

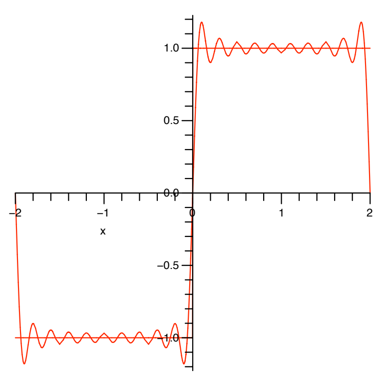


Graphs for Homework §10.2

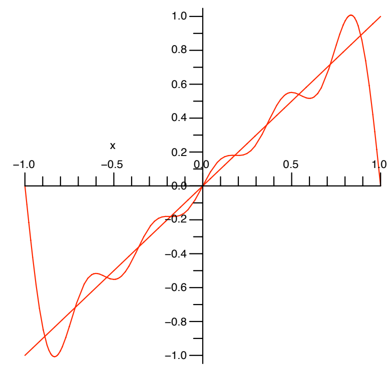
#19(c). The graphs of $s_m(x)$ for $m = 5, 10$, and 20, each with the graph of $f(x)$.

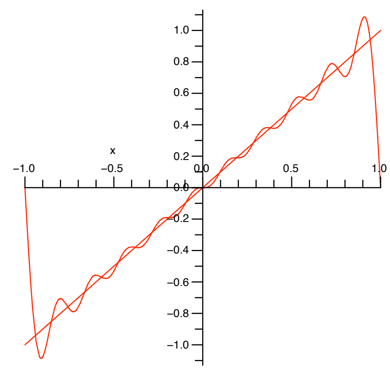


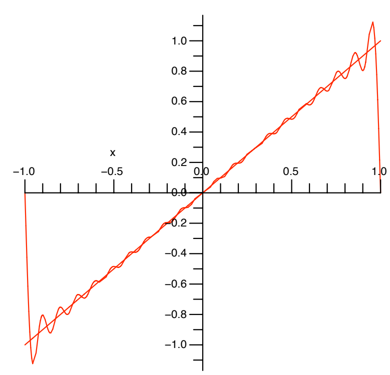




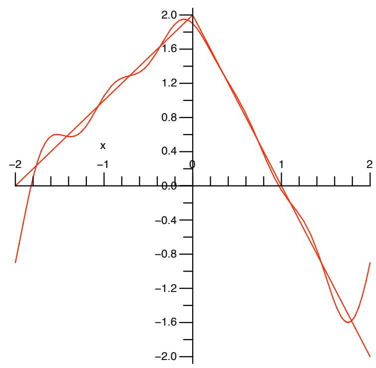
#20(c). The graphs of $s_m(x)$ for $m = 5, 10$, and 20, each with the graph of $f(x)$.

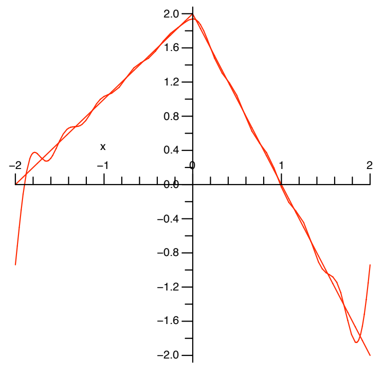


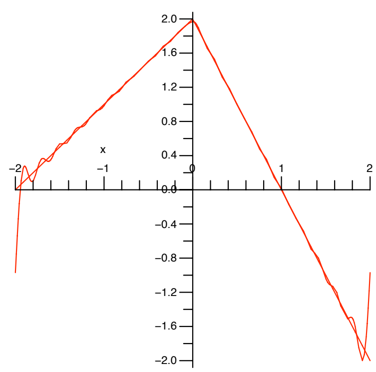




#22(c) The graphs of $s_m(x)$ for $m = 5, 10$, and 20, each with the graph of $f(x)$

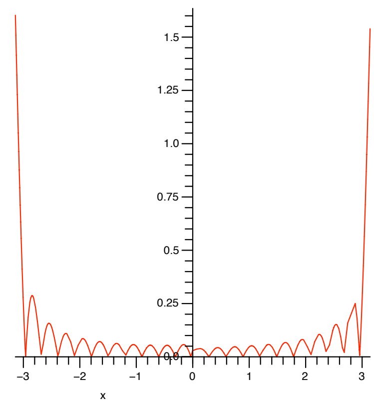


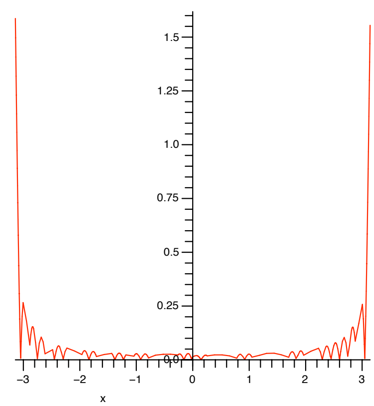


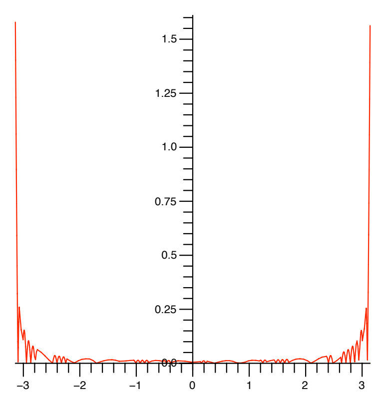


Graphs for Homework §10.3

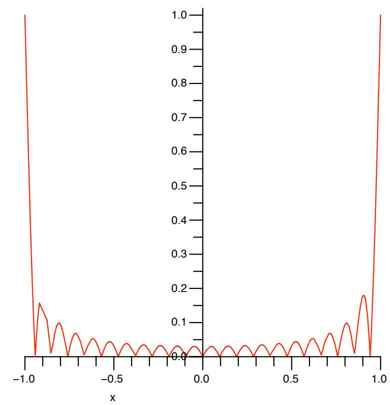
#7(b). The graphs of $e_n(x)$ for $n = 10, 20$, and 40.

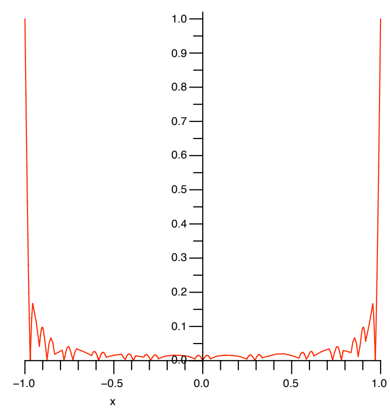


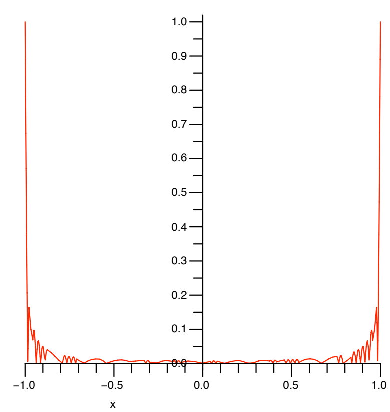




#9(b). The graphs of $e_n(x)$ for $m = 10, 20$, and 40.







#10(b) The graphs of $e_n(x)$ for $m = 10, 20$, and 40.