FOUNDATIONS OF APPLIED MATHEMATICS

Workshop on Teaching Computational Science and Bridging the HPC Talent Gap with Computational Science Research Methods

Emily Evans, Jeffrey Humpherys, Tyler Jarvis
Department of Mathematics
Brigham Young University

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Who are we and what makes us different?

- A group of pure and applied mathematicians with both industry and cross-disciplinary experience.
- We understand that the typical BS is mathematics recipient does not have the skills necessary to compete (or contribute) to computational science.
- Realize that a solid understand of the mathematics behind computational science (and especially computational data science) is extremely helpful.
- We want to develop a program that combines advanced relevant mathematics with computational science.
Opportunities

• Exponential growth in technology has created new opportunities for collecting and storing data.
• The information age has become the information economy
• Algorithms and “big data” are driving innovation
• Just like the DOTCOM boom, everyone and their dog is now a data scientist
• As with every boom, there’s a bust, and the survivors will be those who are really good.
• A knowledge of both computational science and advanced mathematics will differentiate the good.
How did we design an applied and computational mathematics degree from the ground up?
Data Scientist Job @ Amazon.com

Basic Qualifications

• Bachelor's degree in CS, Math, Statistics or a quantitative field
• 3+ years of hands-on experience in predictive modeling and analysis
• 3+ years of hands on experience with one of the statistical software tools: R, Python or SAS
• 3+ years of writing complex SQL queries in a high volume database environment

Preferred Qualifications

• Experience/Knowledge of machine learning techniques such as GBM, random forest etc.
• Experience in e-commerce / on-line companies in fraud / risk control functions
• Coding skills in one of the modern languages Java, Python, Scala
• Experience with visualization technologies such as Tableau
• Experience in statistical techniques such as classification, clustering, regression, statistical inference, collaborative filtering, and natural language processing, experimental design, social networking analysis, feature engineering etc.
• Compelling communication and influencing skills and participation in winning the support of management and influence the course of major strategic decisions
Desired Outcomes

Want the students to

• Become sophisticated mathematically
• Develop strong technical and computational data science skills
• Be engaged and foster a strong work ethic
• Learn to be world class thinkers and problem solvers
• To become leaders and work well in interdisciplinary teams
• To get top career and graduate program placements
• Forge strong bonds that translate into vibrant alumni relations

Want the program to

• Develop tight relationships with industry
• Seed other disciplines with exceptional graduate students
• Seed mathematics with exceptional graduate students
• Help mathematics align better with industry and the needs to society
• Provide a national model that elevates mathematics in the STEM community
• Connects faculty and practitioners to other disciplines and resources
• Change the culture of mathematics
A 21\textsuperscript{st} Century math degree should include…

- Algorithms and Analysis
  - Data Structures
  - Approximation Theory
  - Numerical Analysis
  - Optimization (!)
  - Distributed Computing and Big Data (MPI, Hadoop, noSQL)

- Mathematical and Statistical Modeling
  - Data Analytics (Regression, Estimation, SQL, R/Python)
  - Modeling with Probability and Stochastic Processes
  - Bayesian Statistics
  - Machine Learning
  - Dynamical Systems (ODE, PDE, SDE)
  - Calculus of Variations and Optimal Control
Key Program Features

- New & Modernized Curriculum
  - Cuts through the jargon of various disciplines
  - Develops a rigorous foundation in mathematics, statistics
  - Gain strong technical skills in both computation and data analytics
- Horizontal Integration Across Multiple Quantitative Disciplines
  - Gives students a broad exposure to several interdisciplinary fields
  - Allows each student to have a primary area of specialization.
- Leadership & Soft-Skills Training
  - Foster socialization and team-building amongst groups of students
  - Networking opportunities with scholars and industry leaders
- Capstone Experience
  - Either undergraduate research or an internship
  - Offers rich opportunities for growth outside of the classroom.
  - Launch into next stage of education or career.
Additional Components

- Lock step approach (cohorts)
- Every course has a computer lab to help teach algorithms, high-performance computing, and big-data problems
- Integrates research and mentoring into the curriculum
- Continually connecting mathematics, statistics, and computation with interdisciplinary fields
Program Overview

• Freshman & Sophomore Years
  • General Education Requirements
  • Minor in Mathematics (3 Calculus, Linear Algebra, Introduction to proofs, Differential Equations)
  • Intro Computer Programming (C++)
  • First Semester of Real Analysis

• Junior Year
  • Mathematical Analysis
  • Design, Analysis & Optimization of Algorithms
  • Work on Concentration

• Senior Year
  • Modeling w/ Uncertainty & Data
  • Modeling w/ Dynamics and Control
  • Work in Concentration
Growing list of Concentrations

- Animation
- Biology
- Business Management
- Business Strategy
- Chemical Engineering
- Chemistry
- Civil Engineering: Geotechnical
- Civil Engineering: Structures and structural mechanics
- Civil Engineering: Transportation
- Civil Engineering: Water Resources and Environmental
- Computer Science
- Economics
- Electrical and Computer Engineering: Circuits
- Electrical and Computer Engineering: Electromagnetics
- Electrical and Computer Engineering: Signals and Systems
- Financial Markets
- Geological Sciences
- Linguistics
- Manufacturing Systems Design
- Mathematical Biology
- Mathematical Theory
- Mechanical Engineering: Dynamic Systems
- Mechanical Engineering: Fluids and Thermodynamics
- Physics
- Political Science
- Statistics
- Statistics: Actuarial Science
- Statistics: Biostatistics
Leadership and Soft-Skills Training

- Resumes
- Cover Letters
- Interviews
- Internships
- How to give a talk
- Personality Theory
- Listening
- Conflict Management
- Negotiation
- Leadership
- Running a Meeting
- Project Management
- Working in Teams
- Networking
Partial List of Internships

- Amazon
- Goldman Sachs
- Google
- Facebook
- Microsoft
- Apple
- Raytheon
- Lawrence Livermore
- FBI
- Medic Life
- EPIC
- Rincon
- Ancestry

- Federal Reserve (NY)
- Lincoln Labs
- Sandia NL
- PG&E
- Lucid
- Fast Enterprises
- Los Alamos
- NASA
- Intermountain Health
- UnitedHealth
- Echostar
- Bates White
- Dept Homeland Security
Progress So Far

- **Size of Junior Core**
  - 15 Graduated April 2015
  - 25 Graduated April 2016
  - 31 Graduated April 2017
  - 42 Will be in the Senior Core in the Fall
- **Won 3 of the last 5 ACM regional coding competitions**
- **Excellent job and graduate school placements**
For more information

- Lab Manuals
  - 96 computing labs

- Soft Skills
  - Slides

- Supporting Materials
  - Slides

- 4 Books: To be published by SIAM
  - Foundations of Applied Mathematics
    - Volume 1 Math Analysis
    - Volume 2 Algorithms, Approximation, and Optimization
    - Volume 3 Modeling with Uncertainty and Data
    - Volume 4 Modeling with Dynamics and Control

github.com/Foundations-of-Applied-Mathematics
For more information
see ACME.BYU.EDU
# First Year Sequences

## Mathematical Analysis
- Vector Spaces
- Linear Transformations
- Inner Product Spaces
- Spectral Theory
- Metric Topology
- Differentiation
- Contraction Mappings
- Integration
- Integration on Manifolds
- Complex Analysis
- Advanced Spectral Theory
- Krylov Subspaces
- Pseudospectrum

## Algorithm Design & Optimization
- Intro Algorithms
- Graph Algorithms
- Discrete Probability
- Fourier Theory
- Wavelets
- Interpolation
- Unconstrained Optimization
- Convex Analysis
- Linear Optimization
- Nonlinear Optimization
- Dynamic Optimization
- Markov Decision Processes
First Year Labs

Mathematical Analysis

- Intro Python
- NumPy
- MatPlotLib
- Complexity/Sparse Matrices
- Linear Systems
- QR (householder)
- QR (givens)
- Markov Chain Lab
- Image Segmentation
- Facial Recognition (SVD)
- Finite Differences
- Conditioning
- Newton Cotes vs. Monte Carlo
- Sparse Grid Approximation
- Variance Reduction Methods
- Complex Analysis
- Profiling and Wrapping
- PageRank
- Arnoldi Iteration and GMRES
- The Pseudospectrum

Algorithm Design & Optimization

- Standard Library
- Object Oriented Programming
- Data Structures
- Depth/Breadth First
- Nearest Neighbor Search
- Scientific Visualization
- Maximum Likelihood Estimation
- FFT and Applications
- Wavelets
- Chebychev Polynomials
- Gaussian Quadrature
- Polynomial Interpolation
- Optimization Packages
- Line Search Methods
- Conjugate Gradient Methods
- Simplex Method
- Compressed Sensing Lab
- Interior Point Methods
- Dynamic Optimization
- Multi-Armed Bandits
Second Year Sequences

Modeling with Uncertainty & Data
- Random Spaces & Variables
- Distributions & Expectation
- Limit Theorems
- Markov Processes
- Poisson, Queuing, Renewal
- Information Theory
- Kalman Filtering & Time-Series
- Principal Components
- Clustering
- Bayesian Statistics (MCMC)
- Logistic Regression
- Random Forests
- Support Vector Machines
- Deep Neural Networks

Modeling with Dynamics & Control
- ODE Existence & Uniqueness
- Linear ODE
- Nonlinear Stability
- Boundary-Value Problems
- Hyperbolic PDE
- Parabolic PDE
- Elliptic PDE
- Calculus of Variations
- Optimal Control
- Stochastic Control
Second Year Labs

Modeling with Uncertainty & Data

- Relational Databases and SQL I
- Relational Databases and SQL II
- Regular Expressions
- Scraping with BeautifulSoup
- MPI and OpenMP
- Pandas I
- Pandas II
- Pandas III
- Pandas IV
- Hadoop
- Spark
- JSON, XML
- Web Servers
- Ipython parallel
- MongoDB
- noSQL
- Bokeh

Modeling with Dynamics & Control

- Harmonic Oscillators and Resonance
- Weightloss Models
- Predator-Prey Models
- Shooting Methods and Applications
- Compartmental Models (SIR)
- Pseudospectral methods for BVP
- Lyapunov Exponents and Lorenz Attractors
- Hysteresis in population models
- Conservation Laws and Heat Flow
- Anisotropic diffusion
- Poisson equation, finite difference
- Nonlinear Waves
- Finite Volume Methods
- Finite Element Methods
- Scattering Problems
- PID Control
- LQR and LQG Control
- Guided Missiles
- Merton Model in Finance