Chapter 1 Quiz, Math 112, Section 9, Fall 2003

Please write your name in the upper right corner of this page. No calculators, no people, no books, and no notes (just like if you took it in class). Show your work.

1. Find the least upper bound of the following sets, if it exists.
   (a) \( \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0, p < 7 \right\} \) (Hint, what happens if \( q = -1 \) and \( p \) is a large negative number)
   (b) \( \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0, |p| < 3 \right\} \)
   (c) \((\infty, 4)\)
   (d) \((4, \infty)\)
   (e) \((0, 3] \cap [3, 7)\)

2. Give the domain of each of the following functions.
   (a) \( f(x) = \frac{1}{\sqrt{x + 3}} \)
   (b) \( f(x) = \sqrt{x^2 - 4} \)

3. Find equations of the following lines.
   (a) The line perpendicular to the line \( 3y + 4x = 4 \) which passes through the point \((0,2)\).
   (b) The line which passes through the points \((1, 0)\) and \((3, 3)\).

4. The population of a certain colony was 1,000 in 1870, and was 43,000 in 1900. Assuming the population was growing exponentially, find an exponential function modeling the population as a function of time.

5. Find \( \cos(\frac{\pi}{12}) \) by writing \( \frac{\pi}{12} \) in terms of \( \frac{\pi}{3} \) and \( \frac{\pi}{4} \).

6. Find the coordinates of the maximum of the function
   \[ f(x) = -(3x - 9)^2 + 3 \]

7. Find the inverse of \( f(x) = \frac{x}{5x - 1} \). Give the domain and range of both \( f(x) \) and \( f^{-1}(x) \).

8. Prove \( 1 + \cot^2(x) = \csc^2(x) \).

9. Find all real solutions to \( \sin 2x = \cos x \) (Hint, use the double angle or addition formula for sine to simplify).

10. A certain radioactive substance decays from 35 grams to 17 grams in 8 days. What is its half-life?