Abstract. Test covering section 6.1 through 6.7.

1. (10 pts) Evaluate \[ \int_0^{\pi/3} \sec(x) \tan(x) \, dx. \]

\[ 1 \]

2. (10 pts) Evaluate \[ \int_1^2 \frac{2x^3 - x^2 + 15}{x} \, dx. \]

\[ \frac{19}{6} + 15 \ln 2 \]

3. (10 pts) Find \[ \int \frac{e^{1/x^3}}{x^4} \, dx. \]

\[ -\frac{1}{3} e^{1/x^3} + C \]
4. (10 pts) Evaluate \( \int_0^2 \sqrt{4 - x^2} \, dx \). (Hint: use an area argument)
\[ \pi \]

5. (10 pts) Evaluate \( \int_0^1 \frac{x}{x + 1} \, dx \).
\[ 1 - \ln 2 \]

6. (10 pts) Find \( \int \cosh x + \sinh x \, dx \).
\[ \sinh x + \cosh x + C \]

7. (10 pts) Find \( \int x^2 \sqrt{x + 1} \, dx \).
\[ \frac{2}{7} (x + 1)^{7/2} - \frac{4}{5} (x + 1)^{5/2} + \frac{2}{3} (x + 1)^{3/2} + C \]

8. (10 pts) Find \( \int \csc^2 x \, dx \)
\[ - \cot x + C \]
9. (10 pts) Evaluate \( \int_{\pi/2}^{\pi} \frac{\cos(x)}{1 + \sin(x)} \, dx \).
   \(- \ln 2\)

10. (10 pts) Find \( \int x^2 \sin(15x^3 + 2) \, dx \).
    \(- \frac{1}{45} \cos(15x^3 + 2) + C\)

11. (10 pts) Find \( \int \frac{dx}{4 + x^2} \).
    \(\frac{1}{2} \arctan \left( \frac{x}{2} \right) + C\)

12. (10 pts) Find \( \frac{dA}{dx} \) for \( A(x) = \int_{0}^{\arcsin(x)} e^{-t^2} \, dt \).
    \(e^{-\arcsin^2 x} \frac{x}{\sqrt{1 - x^2}}\)

13. (10 pts) Find \( \frac{d}{dx} \left( \int_{x}^{x^2} \sqrt{t^3 + 2} \, dt \right) \).
    \(2x\sqrt{x^6 + 2} - \sqrt{x^3 + 2}\)
14. (10 pts) Do the following:
   (a) Approximate \( \int_{0}^{\pi/2} \sin^2(x) \, dx \) by computing \( \text{RHS}(3) \).

   \[ \frac{\pi}{3} \]

   (b) Is \( \text{RHS}(3) \) an under-estimate or an over-estimate of the integral above?
   (Hint: The monotonicity of a function is given by the sign of the first derivative.)

   Over-estimate

15. (10 pts) Find the average value of the function \( f(x) = \frac{\ln x}{x} \) on the interval \((1, e)\)

   \[ \frac{1}{2(e - 1)} \]

16. (10 pts) Let \( I = \int_{-1}^{3} f(x) \, dx \), \( J = \int_{-1}^{1} f(x) \, dx \) and \( K = \int_{-3}^{3} f(x) \, dx \). Express the following in terms of \( I, J, \) and \( K \).

   \[ 2(K - I) - 5(I - J) = 2K - 7I + 5J \]

17. (10 pts) A particle has velocity \( v(t) = 3t^2 - 2t \), where \( t \) is measured in seconds. How far did the particle travel between the times \( t = 2 \) and \( t = 5 \)?

   96 units

18. (10 pts) A function \( f(x) \) is concave up and increasing on an interval \([a, b]\). The area underneath \( f \) is \( A \). Arrange \( \text{TRAP}(n), \text{MPS}(n), \text{LHS}(n), \text{RHS}(n), \) and \( A \) in order from smallest to largest.

   \[ \text{LHS, MPS, A, TRAP, RHS} \]