MATH 112 SOLUTIONS FOR 2.2, P.100

1. (a) about 1.2. (b) about –1. (c) 0 (d) 0 (e) 0 (f) about $\frac{1}{2}$. (g) 1 (h) DNE
2. (a) The limit is 5 because the line $y = 2x - 3$ passes through the point (4, 5). (b) 6. (c) 1. (d) –1. (f) –5. (g) 1. (h) DNE. (j) DNE. (l) 0. (n) 0. (p) DNE.
3. (a) $|(2x-1)-5| < .001 \Rightarrow 2|x-3| < .001 \Rightarrow |x-3| < .0005$.

   (b) The graph of $y = \frac{1}{x}$ is between .499 and .501 for $1.998 < x < 2.002$.

   (c) $|(x^2+1)-5| = |x^2-4| = |x+2||x-2|$. If $|x-2| < 1$, then $x < 3$ and $|x+2| < 5$. If$|x-2| < \delta$, then$|x+2||x-2| < 5\delta$, so let $5\delta < .001$, or $\delta < .0002$.

   (d) $|\sqrt{x}-4| = \dfrac{|x-16|}{|\sqrt{x}+4|}$. If $|x-16| < 1$, then $x > 15$ and $|\sqrt{x}+4| > 4+\sqrt{15}$. If also $|x-16| < \delta$, then $|\sqrt{x}-4| = \dfrac{|x-16|}{|\sqrt{x}+4|} < \dfrac{\delta}{4+\sqrt{15}}$, so let $\dfrac{\delta}{4+\sqrt{15}} < .001$. Then $\delta < (4+\sqrt{15})(.001) \approx .00787$.
4. (b) Since $f(\frac{1}{2}) \approx .18877$, plot $f(x) = \dfrac{x}{e^x+1}$ in the window $[.495, .505] \times [.18777, .18977]$. The graph appears between $x \approx .4962$ and $x \approx .5039$. Hence we may choose $\delta < .003$.
5. (a) Let $\delta = .0005$. Then $|x-3| < \delta \Rightarrow |(2x-1)-5| = 2|x-3| < 2\delta = .001$.

   (c) $|(x^2-1)-8| = |x^2-9| = |x+3||x-3|$. If $|x-3| < 1$, then $|x+3| < 7$. Let $\delta = \frac{.004}{7} \approx .0005$. Then $0 < |x-3| < \delta \Rightarrow |x-3||x+3| < (.0005)(7) = .0035 < .004$.
6. (a) Given $\epsilon > 0$, let $\delta = \frac{\epsilon}{3}$. Then $0 < |x-2| < \delta \Rightarrow |(3x+1)-7| = 3|x-2| < 3 \cdot \frac{\epsilon}{3} = \epsilon$.

   (b) Let $\epsilon > 0$ be given, and let $\delta < 6\epsilon$. Then $|x-9| < \delta \Rightarrow |\sqrt{x}-3| = \dfrac{|x-9|}{|\sqrt{x}+3|} < \dfrac{\delta}{3+\sqrt{9-\delta}} < \dfrac{\delta}{6} < \epsilon$.

   (c) $|(x^2+4)-5| = |x^2-1| = |x-1||x+1|$. If $|x-1| < 1$, then $|x+3| < 3$. Hence, given $\epsilon > 0$, let $\delta = \min\{1, \frac{\epsilon}{3}\}$. Then $0 < |x-1| < \delta \Rightarrow |(x^2+4)-5| = |x-1||x+1| < \frac{\epsilon}{3} \cdot 3 = \epsilon$.

   (d) $|\frac{2}{x} - (-2)| = \frac{2|x+1|}{|x|}$. Let $|x+1| < \frac{1}{2}$. Then $-\frac{1}{2} < x+1 < \frac{1}{2} \Rightarrow -\frac{3}{2} < x < -\frac{1}{2}$, so $|x| > \frac{1}{2}$ and $\frac{1}{|x|} < 2$. Now given $\epsilon > 0$, let $\delta = \min\{\frac{1}{2}, \frac{\epsilon}{4}\}$. Then $0 < |x-(-1)| < \delta \Rightarrow |\frac{2}{x} - (-2)| = 2|x+1| \cdot \frac{1}{|x|} < 2 \cdot \frac{\epsilon}{4} \cdot 2 = \epsilon$.
11. Suppose both $f(x) \to L$ and $f(x) \to M$ as $x \to c$, with $L \neq M$. Let $\epsilon = \frac{1}{4}|L-M|$. Then there exists δ_1 such that $|x-c| < \delta_1 \Rightarrow |f(x)-L| < \epsilon$ and there exists δ_2 such that $|x-c| < \delta_2 \Rightarrow |f(x)-M| < \epsilon$. Hence, if $\delta = \min\{\delta_1, \delta_2\}$, then $|x-c| < \delta \Rightarrow |L-M| = |[L-f(x)] + [f(x)-M]| \leq |f(x)-L| + |f(x)-M| < 2\epsilon = \frac{1}{2}|L-M|$, a contradiction.
14. (a) $\frac{1}{2}$. (b) $\frac{1}{2}$. (c) $\frac{1}{2}$. (d) –1. (e) 1. (f) DNE. (g) DNE. (h) DNE. (i) DNE. (j) 0. (k) 0. (l) 0.
16. (b), (c), and (d) are true.

17. For example, if $x_1 = 1$, then $x_2 = 1.5, x_3 = 1.41\overline{6}, x_4 = 1.4142157, x_5 = 1.4142136, x_6 = 1.4142136, \ldots$. $g(x) = x \Rightarrow x = \pm\sqrt{2}$; $\sqrt{2} \approx 1.4142136$.