MATH 112 SOLUTIONS FOR 2.6, P. 167

1. (a) about –1.97. (b) $f'(1) = -2$. (c) $f'(x) = \dfrac{-2}{x^3}$, so $f'(1) = -2$. (d) $y = 3 - 2x$.

2. (c) Assume that the function has value 0 at $x = 0$. The graph never straightens out, indicating nondifferentiability at the origin.

    (d) Assume the function has value 0 at $x = 0$. The graph eventually flattens out, indicating that the derivative at 0 is 0.

3. (c) $\frac{1}{9}$. (e) $f'(0) = 1$.

4. (b) $f'(x) = 2ax + b$. (c) $-1/x^2$.

5. (a) (i) The one-sided limits of f at 1 are both 1= $f(1)$, so f is continuous at 1. (ii) $f'(1) = \lim_{b \to 1} \dfrac{f(b) - f(1)}{b - 1}$. The two one-sided limits of this expression are both 2, so $f'(1) = 2$.

    (b) (i) The one-sided limits are not equal, so the function is not continuous at 1. Since it is not continuous at 1, it is not differentiable at 1.

    (c) (i) The one-sided limits at –1 both have value 5, so f is continuous at 5. (ii) $f'(-1) = \lim_{x \to -1} \dfrac{f(x) - f(-1)}{x + 1}$. The two one-sided limits of this expression are both –2, so f is differentiable at –1 and the derivative is –2.

6. Assume that f, g, and h all have value 0 at 0.

    (a) $\lim_{x \to 0} \sin \frac{1}{x}$ does not exist, so f is not continuous at 0.

    (b) $\lim_{x \to 0} x \sin \frac{1}{x} = 0 = g(0)$, so g is continuous at 0. But $g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \sin \frac{1}{x}$, which does not exist, so g is not differentiable at 0.

    (c) $\lim_{x \to 0} h(x) = 0 = h(0)$, so h is continuous at 0. $h'(0) = \lim_{x \to 0} \frac{h(x) - h(0)}{x - 0} = \lim_{x \to 0} x \sin \frac{1}{x} = 0$, so h is differentiable at 0.

11. $\dfrac{dA}{dr} = 2\pi r$

14. $\dfrac{dR}{dq} = 460 - .04q$ and $\left.\dfrac{dR}{dq}\right|_{q=300} = 448$.

17. Derivative is slope, so the derivative of a linear function is the (constant) slope of its graph.

20. (a) .33 (c) .12, .23, and .175