MATH 112 SOLUTIONS FOR 3.2, P. 237

1. (a) none. (c) 0, $\frac{3}{4}$. (d) $-\dfrac{1}{2^{1/3}}$. (e) o, 3, –3. (g) 1. (i) $\frac{\pi}{4}+n\pi, n \in \mathbb{Z}$. (k) $\pi + 2n\pi, n \in \mathbb{Z}$. (o) 0.
2. (a) increasing and concave upward. (b) concave downward. (c) decreasing and concave upward. (d) increasing and concave downward. (e) decreasing and concave downward.
6. Since f is of odd degree, it has at least one real zero. If f has two real zeros, then f has a critical point, by Rolle's Theorem. But $f'(x) = 3x^2 + 4 > 0$ for every x, and f has no critical point, so at most one real zero.
8. (c) $\tan 0 \neq \tan \pi/3$
9. Let $f(t), g(t)$ be the positions of the cars, and we assume f and g are differentiable. Let $h(t) = f(t) - g(t)$. Since each car passes the other, there are two times t_1 and t_2 at which they have the same position, so $h(t_1) = h(t_2) = 0$. Hence by Rolle's Theorem there is a time T between t_1 and t_2 such that $h'(T) = 0$, so that $f'(T) = g'(T)$ and the cars have the same speed at time T.
10. Since the cars pass each other, there is a time T_1 at which their speeds are the same; when they pass each other again, there is a time T_2 at which their speeds are the same. Hence there is a time between T_1 and T_2 when the derivatives of their speeds, their accelerations, are the same.
11. (a) $c = 1$ (c) $c = \ln(\frac{e^e-1}{e}) \approx 1.65002$ (d) $c = \dfrac{1}{\ln(1+e)} \approx 0.31326$.
12. (d) $x^{2/3}$ is not differentiable at 0.
13. (c) cp: 0, 2. hcp: 1. increasing in $(-\infty, 0)$ and in $(2, \infty)$. decreasing in $(0, 2)$. concave down in $(-\infty, 1)$. concave up in $(1, \infty)$. infl.pt. at 1

    (g) cp: –1, 1. hcp: none (0 is not in the domain). increasing in $(-\infty, -1)$ and in $(1, \infty)$. decreasing in (–1, 0) and in (0, 1). concave down in $(-\infty, 0)$. concave up in $(0, \infty)$. infl.pt.: none.

    (h) cp: 0. increasing in $(-\infty, 0)$, decreasing in $(0, \infty)$. hcp: $\pm\frac{1}{\sqrt{3}}$. concave down in $(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, concave up in $(-\infty, \frac{-1}{\sqrt{3}})$ and in $(\frac{1}{\sqrt{3}}, \infty)$.

    (l) cp: none. increasing everywhere. hcp: $n\pi$. concave down in $(\pi + 2n\pi, 2\pi + 2n\pi)$, concave up in $(2n\pi, \pi + 2n\pi)$.

    (n) cp: 1. increasing in $(-\infty, 1)$, decreasing in $(1, \infty)$. hcp: 2. concave down in $(-\infty, 2)$, concave up in $(2, \infty)$.

    (o) cp: 0. hcp: –1, 1. increasing in $(-\infty, 0)$. decreasing in $(0, \infty)$. concave up in $(-\infty, -1)$ and in $(1, \infty)$. concave down in (–1, 1). infl.pt.: -1, 1.
14. (c) vertical tangent line at 0.
15. If $f''(x) < 0$, then f' is decreasing by Theorem 71, so f is concave downward.
16. (a) $y = e^a x + (1-a)e^a$. (b) $(1-a)e^a$. (c) The curve is concave upward, so the tangent line, and hence its intercept, is always below the curve.

17. (a) If $p(c_1) = p(c_2)$, then by Rolle's Theorem there is a point between c_1 and c_2 at which $p'(x) = 0$. But then x is a critical point, a contradiction.
19. (c) cusp. (d) vertical tangent line.
21. (a) The curve is concave downward because as the temperature increases, the rate of temperature increase lessens. (b) $60 \leq T(35) \leq 72$. (c) $68 \leq T(35) \leq 72$. (d) between $t = 32.5$ and $t = 38.75$.
22. Apply the Mean Value Theorem to f on the interval $[a, x]$. Thus there exists $c \in (a, x)$ such that $f'(c) = \dfrac{f(x) - f(a)}{x - a}$, from which the result follows.