MATH 112 SOLUTIONS FOR 3.3, P. 248

1. (b) global minimum at $(-1, 2)$. (f) global maximum at $\left(\dfrac{1}{\sqrt[3]{2}}, \dfrac{1}{4}\right)$. (j) global maximum at $(3, \frac{8}{3})$. (n) global minimum at the origin.
2. (b) global maximum at $\left(\dfrac{1}{\sqrt{2}}, \sqrt{2}\right)$, global minimum at $(-1, -1)$, and local minimum at $(1, 1)$.

   (c) global maximum of $\sqrt{2}$ at $\frac{\pi}{4} + 2n\pi$, global minimum of $-\sqrt{2}$ at $\frac{5\pi}{4} + 2n\pi, n \in \mathbb{Z}$.

   (f) local maximum at $(-\frac{\pi}{3} + 2n\pi, -\frac{\pi}{3} + 2n\pi + \sqrt{3})$, local minimum at $(\frac{\pi}{3} + 2n\pi, \frac{\pi}{3} + 2n\pi - \sqrt{3})$.

   (j) global minimum at $(\frac{1}{3}, -\frac{2}{3e})$.

   (n) no extrema
3. The function $f(x) = \begin{cases} x - a, & a \le x < \frac{1}{2}(a+b) \\ -\frac{1}{2}(x-b), & \frac{1}{2}(a+b) \le x \le b \end{cases}$ Is defined on $[a, b]$ but has no maximum there.
4. f'' changes sign at c, so $(f')'$ changes sign at c and hence f' has an extremum.
5. Theorem. If $f''(c) = 0$ and $f'''(c) \neq 0$, then f has an inflection point at c.

   Proof. If $f''(c) = 0$, then c is a critical point of f' at which the derivative is zero, so we may apply the second derivative test to f'. If $(f')''(c) \neq 0$, then f' has an extremum at c, so that the derivative of f' changes at c and hence the concavity changes at c. Thus c is an inflection point of f.
6. Since f is continuous on a closed interval, it has a global maximum there, which is also a local maximum, so occurs at c.
7. Suppose f is defined in (a, b) and $c \in (a, b)$ is the only critical point of f, and f has a local maximum at c (the case of minimum is similar). Since f is continuous and c is the only critical point, f is differentiable in (a, b) except possibly at c. If f were to have a value greater than $f(c)$ somewhere, then it must have the same value at some point, and then Rolle's Theorem would give us another critical point, which we can't have. Thus the value at c is a global maximum.
8. (a) P2 should maximize S with respect to y, getting $y = \dfrac{27 - x}{4}$.

   (b) P1 should replace y in S with the value that P2 is going to use, and minimize the resulting S, getting $x = -1$.

   (c) P1 should minimize S with respect to x, getting $x = \dfrac{y - 9}{2}$.

   (d) P2 should replace x in S with the value that P1 is going to use and maximize the resulting S, getting $y = 7$.

   (e) Each player has to "go first" so P1 chooses $x = -1$ and P2 chooses $y = 7$. (The resulting score is 0, so the game is uninteresting, even though it is fair.)