MATH 112 SOLUTIONS FOR 3.4, P. 258

1. Maximize $V = x(8-2x)(15-2x)$.
2. Cut out a square of side $\dfrac{s}{6}$ from each corner.
3. Maximize $A = w(400-2w)$, getting a maximum area of 20,000 sq. yd.
4. 25 feet by 10 feet
5. Maximize $A = l(\frac{800-5l}{4})$.
7. Maximize $A = l(\frac{1}{2}p - l)$ to get $l = \frac{1}{4}p$.
8. (a) The triangle will have a maximum altitude when the vertex is on the diameter perpendicular to the chord. That makes the triangle isosceles.

   (b) If the triangle has maximum area, it is isosceles. Let the base chord be a distance x from the center of the circle. Then the altitude of the triangle is $r+x$ and the base is $2\sqrt{r^2-x^2}$. The area is then $A = (r+x)\sqrt{r^2-x^2}$, which has a critical point ar $x = \frac{r}{2}$. It follows that each side of the triangle is $r\sqrt{3}$, so it is equilateral. The area is $\frac{3\sqrt{3}}{4}r^2$.
9. Maximize $P = n(50-n)$.
10. If the vertex of the rectangle in the first quadrant has coordinates $(x, 4-x^2)$, then the area is $A = 2x(4-x^2)$ and has its maximum at $x = \frac{2}{\sqrt{3}}$. The maximum area is $\frac{32}{3\sqrt{3}}$.
12. If the trapezoid has altitude h, then its area is $A = Lh + h\sqrt{L^2-h^2}$. A is maximum at $h = \frac{\sqrt{3}}{2}L$, and its maximum value is $\frac{3\sqrt{3}}{4}L^2$.
13. Maximize $V = \pi(\sqrt{r^2-h^2})^2 h$.
18. Let the can have height h and base radius r, with given volume V. The amount of material used is the same as the total area, which is $A = 2\pi rh + 2\pi r^2$. Using $V = \pi r^2 h$, we eliminate h and get $A = \frac{2V}{r} + 2\pi r^2$, which has its minimum at $r = (\frac{V}{2\pi})^{1/3}$. It follows that $h = 2r$.
20. If R is the rent, then the profit is $P = (R-40)(80 - \frac{R-400}{20}) - 24000$, which has maximum at $R = \$1020$.
23. We set $f'(P) = 1$ and find that the maximum sustainable removal occurs at $P = 12$. That is, fishermen may remove up to 1200 fish each year without depleting the fish population.
24. Let α be the angle of elevation to the bottom of the tapestry and let θ be the angle subtended by the tapestry at the eye. Then $\alpha + \theta$ is the angle of elevation to the top of the tapestry. If the observer is standing a distance x from the wall on which the tapestry is hanging, then $\tan\alpha = \frac{2}{x}$ and $\tan(\alpha+\theta) = \frac{16}{x}$, from which we find $\theta = \tan^{-1}\frac{16}{x} - \tan^{-1}\frac{2}{x}$. The maximum of θ occurs at $x = 4\sqrt{2}$ feet.
25. A rod that touches the inside corner of the turn and both outside walls has length $l = x + \dfrac{4x}{\sqrt{x^2-16}}$, where x is the length of the portion of the rod from the corner to the wall. The minimum such length l is the length of the longest rod that will go around the turn without bending. The minimum l is $8\sqrt{2}$. (Note that letting $x = l - x$ gives the same solution.)

26. Minimize $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + 4 - x}$. The minimum D occurs at $x = \frac{1}{2}, y = \sqrt{\frac{7}{2}}$.
27. Minimize the distance function $D = \sqrt{x^2 + y^2}$, replacing y by $\frac{x+1}{x} = 1 + \frac{1}{x}$. The critical points satisfy the equation $x^4 - x - 1 = 0$; the solution $x \approx -0.7245$ corresponds to $y \approx -0.3803$ and gives the minimum distance $D \approx 0.8182$.
28. Minimize $L = \dfrac{6}{x^2} + \dfrac{1}{(B-x)^2}$ to get $x \approx .65B$, with B the length of the block. That is, the dimmest point is about 65% of the length of the block from the brighter light.
30. Let the ladder of length L rest against the wall at height y and let its foot be distance x from the fence. If the ladder just clears the fence, then $\frac{x}{6} = \frac{x+2}{y}$, which we can use to express the length of the ladder, $L = \sqrt{(x+2)^2 + y^2}$, in terms of just one variable. Using $y = \frac{6(x+2)}{x}$, we get $L = \frac{x+2}{x}\sqrt{x^2 + 36}$. The minimum occurs at $x = (72)^{1/3}$, and is $6(1 + 9^{-1/3})^{3/2} \approx 10.81$ ft.
32. (a) Let the ranches be at points A and B, with point C on the river closest to A and point D on the river closest to B. We are given $AC = 16$, $BD = 10$, and $AB = 10$, from which it follows that $CD = 8$. If we let the pumping station be at distance x from C, then the total length of the pipelines is $L = \sqrt{x^2 + 256} + \sqrt{(8-x)^2 + 100}$. This function has minimum at $x = \frac{64}{13}$.
    (b) The pipeline would run from D to B to A.
    (c) Let B' be the reflection of B in the river; then the shortest pipeline would have the same length as the straight line from A to B', which meets the river at point x between C and D. We can now use similar triangles to find x, which is the same as in (a).