MATH 112 SOLUTIONS FOR 3.7, P. 286

2. $f(x) = x^2 - ax$ describes a parabola opening upward with zeros at 0 and a and vertex $(\frac{a}{2}, -\frac{a^2}{4})$. $g(x) = ax - x^2$ has the same zeros but vertex $(\frac{a}{2}, \frac{a^2}{4})$ and opens downward.
3. Let $f(x) = ax^4 + bx^2 + c$. The critical points are 0 and $\pm\sqrt{\frac{-b}{2a}}$. $f''(0) = 2b$ and $f''(\pm\sqrt{\frac{-b}{2a}}) = -4b$. Hence if $a > 0$ and $b < 0$, f has global minima at $\pm\sqrt{\frac{-b}{2a}}$ and a local maximum at 0. If $a > 0$ and $b > 0$, f has only a global minimum at 0. If $a < 0$ and $b > 0$, f has global maxima at $\pm\sqrt{\frac{-b}{2a}}$ and a local minimum at 0. If $a < 0$ and $b < 0$, f has only a global maximum at 0.
4. Let $f(x) = ax^3+bx^2+cx+d$. The critical points are at $x = \dfrac{-b \pm \sqrt{b^2 - 3ac}}{3a}$ and the inflection point is at $x = \dfrac{-b}{3a}$. The average of the x-coordinates of the critical points is clearly the x-coordinate of the inflection point. The same is true for the y-coordinates.
5. If the cubic in Problem 4 has only one critical point, it is because $b^2-3ac < 0$ and the critical point is $x = \dfrac{-b}{3a}$, the inflection point.
8. Let the cubic have the equation $f(x) = ax^3 + bx^2 + cx + d$. If (3, 0) is the local maximum and (5, –1) is the inflection point, then the local minimum is at (7, –2) and a is positive. The inflection point is at $x = \frac{-b}{3a} = 5$, so $b = -15a$. Since the first critical point is $x = \dfrac{-b - \sqrt{b^2 - 3ac}}{3a} = 3$, we get $c = 63a$. Finally, using $f(3) = 0$, we get $d = -81a$. Hence $f(x) = a(x^3 - 15x^2 + 63x - 81)$ with $a > 0$.
10. (a) Since 120 mi/hr = 176 ft/sec, the maximum steepness is $\frac{8}{176} = \frac{1}{22}$, so the minimum slope is $-\frac{1}{22}$.

    (b) With $f(x) = ax^3 + bx^2 + cx + d$, we let the inflection point be the origin; since the inflection point is at $x = \frac{-b}{3a}$, we have $b = 0$. Since $f(0) = 0$, we have $d = 0$. The slope at the inflection point is $f'(0) = c = \frac{-1}{22}$. The critical points are then at $x = \pm\sqrt{\frac{-c}{3a}} = \pm\frac{1}{\sqrt{66a}}$ and $y = \mp\frac{1}{33\sqrt{66a}}$. Therefore the run is $\frac{2}{\sqrt{66a}}$ and the rise is $-\frac{2}{33\sqrt{66a}}$.

    (c) If the rise is –12,000 feet, then the run is $\frac{2}{\sqrt{66a}} = (33)(12000)$ feet = 75 miles.

    (d) From part (c) we have $a = \frac{1}{66(33)^2(6000)^2} \approx 3.86 \cdot 10^{-13}$.
11. Increasing a magnifies the graph vertically. Increasing b compresses the graph horizontally.

13. If the maximum of $f(x) = axe^{-bx}$ is (7,55), then $b = \frac{1}{7}$ and $a = \frac{55e}{7}$. It follows that $f(x)$ is less than 20 when $\frac{55e}{7}xe^{-x/7} < 20$, which is true for $x > 22.1422$.
14. $y = 5[1 - (\frac{4}{5})^x]$.
17. $\lim_{x\to\infty} \frac{a}{1+be^{-kx}} = a$ and $\lim_{x\to-\infty} \frac{a}{1+be^{-kx}} = 0$.
20. Demand will be about 24 sales per week, and it will take another four weeks to reach it.
22. about 1557.
25. Think of $y = e^{-ax}$ as an amplitude to a sunisoid.
26. The graph is a bell curve having its maximum at $x = \mu$ and inflection points at $x = \mu \pm \sigma$.
27. Let the chain hang from the points $(-p, q)$ and (p, q) with its low point at the origin. Since the low point is a critical point, we get $f'(0) = 0 = bc\sinh(d)$, so $d = 0$. Since the origin is on the curve, we have $f(0) = 0 = a + b\cosh(0) = a + b$, and $b = -a$. Then $f(p) = q = a - a\cosh(cp)$, so $a = \dfrac{q}{1-\cosh(cp)}$. Thus $f(x) = \dfrac{q(1-\cosh cx)}{1-\cosh cp}$.