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2. (b) $y^4 = \frac{8}{3}x^{3/2}$. (c) $y^3 = 3x^2+1$. (d) $y^2 = 6(x+\frac{1}{3}x^3)-2$. (f) $\tan y = \frac{1}{2}x^2+1$.
3. (a) about 1986 or 1987. (b) about 1983.
4. $\dfrac{dP}{dt} = kP^{2/3}$; $P = (t+10)^3$; $t = 0 \Rightarrow P = 1000$
5. $\dfrac{dC}{dt} = \dfrac{k}{C}$; $C^2 = \frac{1}{2}t$; $C = \frac{1}{2} \Rightarrow t = \frac{1}{2}$ hr; $C = \frac{1}{3} \Rightarrow t = \frac{2}{9}$, so it takes $2 - \frac{2}{9} = \frac{16}{9}$ hr to completely charge.
8. $\dfrac{dh}{dt} = \dfrac{k}{\sqrt{h}}$; $h^{3/2} = \dfrac{5\sqrt{10}}{2}\,t$; $t = 8 \Rightarrow h = 10 \cdot 2^{2/3}$.
9. 10,506
12. resistive force $= \frac{3}{2}v$; $F = ma = 120 - \frac{1}{2}v - \frac{3}{2}v \Rightarrow a = \dfrac{dv}{dt} = \dfrac{g}{400}(60 - v)$; $v = 60 - Ae^{-gt/400}$; maximum v is 60.
13. $a = g - \frac{1}{7}v \Rightarrow v = 7g + Ae^{-t/7} \Rightarrow v \to 7g$ as $t \to \infty$.
14. $\dfrac{dV}{dt} = 576\pi\dfrac{dh}{dt} = -\pi\sqrt{2g}h^{1/2}$; $h = \left(4\sqrt{3} - \dfrac{\sqrt{g}}{576\sqrt{2}}t\right)^2$; $h = 0 \Rightarrow t = 288$ sec.
15. (a) Since g is inversely proportional to the square of the distance x from the center of the earth, the acceleration on the object is $a = -g = -\frac{k}{x^2}$.

    (b) $a = v\frac{dv}{dx} = -\frac{k}{x^2} \Rightarrow v^2 = \frac{2k}{x} + C$; $x = x_0$ and $v = v_0 \Rightarrow C = v_0^2 - \frac{2k}{x_0}$, and the result follows.

    (c) $v > 0 \Rightarrow x \to \infty$ as $t \to \infty$, so that $v^2 \to v_0^2 - \frac{2k}{x_0} > 0 \Rightarrow v_0 > \sqrt{\frac{2k}{x_0}}$.

    (d)$k = 95040$ and $v_0 > 6.93$ mi/sec.
21. (a) i (b) iii (c) v (d) ii (e) iv