Math 112 Test 2 - section 9
Instructor: Jason Grout (Feb 11 – Feb 13)

No calculators.

Part I: Multiple Choice (5 pts each)

Instructions: Circle the correct answer.

1. Let $f(x)$ and $g(x)$ be differentiable functions. Let $f(1) = 8$, $f'(1) = 6$, $g(1) = 2$, and $g'(1) = 1$. What is the derivative of $\frac{f}{g}$ at $x = 1$?

   (a) 0 (b) 1 (c) 2 (d) 5

   Solution. (b)

2. If $h(x) = g(f(x))$, $f(2) = 7$, $f'(2) = -2$, and $g(x) = x^2$, what is $h'(2)$?

   (a) -28 (b) -8 (c) 14 (d) 20

   Solution. (a)

3. Which of the following is false in general?

   (a) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}$

   (b) $\frac{d}{dx}\sin^2 x = 2\sin x\cos x$

   (c) $(fg)' = fg' + f'g$

   (d) $\left(\frac{1}{f}\right)' = \frac{-1}{f^2}f'$

   Solution. (a) (Remember the chain rule)

4. $\frac{d}{dx}\ln|\cos x| =$

   (a) $\frac{1}{|\cos x|}$

   (b) $\frac{1}{\cos x}$

   (c) $-\tan x$

   (d) $-\sin x$

   Solution. (c)

Part II: Short Answer

Instructions: Show your work. Clearly mark your answers.

5. Find the following derivatives (5 pts each)

(a) $\dfrac{d}{dx}\, x^3 + 3x^2 + 5x + 2$
Solution. $3x^2 + 6x + 5$

(b) $\dfrac{d}{dx}\, e^x$
Solution. e^x

(c) $\dfrac{d}{dx}\, \sin(x^2 + 1)$
Solution. $2x\cos(x^2 + 1)$

(d) $\dfrac{d}{dx}\, \cosh x$
Solution. $\sinh x$

(e) $\dfrac{d}{dx}\, 3^{2x+1}$
Solution. $(3^{2x+1})(2)(\ln 3)$

(f) $\dfrac{d}{dx}\, \sqrt{1 + \csc x}$
Solution. $\frac{1}{2}(1 + \csc x)^{-1/2}(-\csc x \cot x)$

(g) $\dfrac{d}{dx}\, x^3 \ln(3x^2 + 2)$
Solution. $3x^2 \ln(3x^2 + 2) + \frac{6x^4}{3x^2+2}$

(h) $\dfrac{d}{dx}\left(\dfrac{\tan x}{x^2}\right)$
Solution. $\frac{x\sec^2 x - 2\tan x}{x^3}$

(i) $\dfrac{d}{dx}\, |4 - x^2|$
Solution. $\frac{4-x^2}{|4-x^2|}(-2x)$

6. (5 pts) Find the equation of the tangent line to the curve $f(x) = e^{2x}$ when $x = \ln 2$.

   Solution. $y = 8(x - \ln 2) + 4$

7. (5 pts) Find the equation of the normal line to the curve $f(x) = x(x - 5)$ at the point $(0, 0)$. (Hint: remember that the normal line is perpendicular to the tangent line.)

   Solution. $y = \frac{x}{5}$

8. (a) (5 pts) State the definition of the derivative $f'(x)$ of the function $f(x)$

   $f'(x) =$

   Solution. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ or $f'(x) = \lim_{b \to x} \dfrac{f(b) - f(x)}{b - x}$

   (b) (5 pts) Use the definition of the derivative to compute $f'(0)$ for the function

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) + (x+1)^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

   Solution.

$$\begin{aligned} f'(0) &= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \to 0} \frac{h^2 \sin(\frac{1}{h}) + (h+1)^2 - 1}{h} \\ &= \lim_{h \to 0} h \sin(\frac{1}{h}) + h + 2 \\ &= 2 \end{aligned}$$

   Notice that $\lim_{h \to 0} h \sin(1/h) = 0$ because it is the product of a function going to zero (h) and a bounded function ($\sin(1/h)$). Notice that we *cannot* just plug in $h = 0$ in this case.

9. (a) (5 pts) Let f be a function that is continuous at a point a. What can we say about the differentiability of f at a?
Solution. Nothing. The function f may or may not be differentiable at a.

(b) (5 pts) Let g be a function that is differentiable at a point a. What can we say about the continuity of g at a?
Solution. The function g must be continuous at a.

10. (5 pts) Use implicit differentiation to find the derivative $\frac{dy}{dx}$ if

$$xy^4 + \cos(y^2) = x^2 + y.$$

Solution. $y' = \dfrac{2x - y^4}{4xy^3 - 2y\sin(y^2) - 1}$