Math 112 Test 3 - section 9
Instructor: Jason Grout
Test dates: Mar 6 - Mar 7. Late day Mar 8. No calculators.

Part I. Multiple Choice

Instructions: Circle the best answer.

1. (5 pts) Let $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 3$. We can see that $f'(1) = 0$. Using the second derivative test, what can we conclude?
   (a) f has a local minimum at $x = 1$.
   (b) f has a local maximum at $x = 1$.
   (c) f has a root at $x = 1$.
   (d) The second derivative test gives no information.
   Solution. Since $f''(1) = 0$, the correct answer is (d).
2. (5 pts) Let $f(x) = -x^3 - 3x^2 + 24x + 4$. Which of the following is true?
   (a) f is concave down on $(-4, 2)$.
   (b) f is concave up on $(-1, \infty)$.
   (c) f is increasing on $(-4, 2)$.
   (d) f is decreasing on $(-1, \infty)$.
   (e) None of the above.
   Solution. (c).
3. (5 pts) Find $\frac{dy}{dt}$ when $x = \pi$, $y = 0$, $\frac{dx}{dt} = 3$, and $y \ln(\cos^2 x) + x^2 = 3y - 3$.
   (a) 0
   (b) $\pi/3$
   (c) $2\pi/3$
   (d) 2π
   (e) None of the above.
   Solution. (d).
4. (5 pts) The radius of a circle changes from 20 ft to 19.9 ft. Using local linearization, approximately how much does the area decrease?
   (a) 2π
   (b) 4π
   (c) 20π
   (d) 40π
   Solution. (b) $dA = 2\pi r dr = 2\pi(20)(.1)$.

Part II Short Answer

Instructions: Show your work. Clearly mark your answers.

5. (5 pts) Use local linearization to approximate $(1.01)^{10}$.

   Solution. $(1.01)^{10} \approx 1 + 10(1)^9(.01) = 1.1$.

6. (5 pts) We know that a polynomial of odd degree must have at least one real zero. Use Rolle's Theorem to prove that the polynomial $f(x) = x^5 + 8x + 7$ has *only* one real zero.

   Solution. $f'(x) = 5x^4 + 8 > 0$. Suppose f had two zeros, a and b. Since f is continuous on $[a, b]$ and differentiable on (a, b), and $f(a) = f(b)$, Rolle's Theorem states that there exists a $c \in (a, b)$ such that $f'(c) = 0$. But $f'(x) > 0$ everywhere. Therefore, f does not have two zeros.

7. (5 pts) Find the x-intercept of the tangent line to the curve $f(x) = x^7 + 3x + 1$ at $x = 1$. (Hint: You can check your answer using the formula for Newton's method.)

   Solution. $x_2 = \frac{1}{2}$

8. (5 pts) Find the critical point(s) of f. Assume that C, a, and b are constants. Assume that $C \neq 0$ and $b \neq 0$.

$$f(x) = Ce^{\frac{-(x-a)^2}{b}}$$

   Solution. $x = a$

9. (8 pts) Find the absolute maximum and absolute minimum of $f(x) = x + \dfrac{16}{x}$ on the interval [1,32]. (Remember that the absolute max and min are y values)

   Solution. Abs max is 32.5 which occurs when $x = 32$; Abs min is 8 which occurs when $x = 4$

10. Find the following limits: (5 pts each)

    (a) $\displaystyle\lim_{x\to\infty} \frac{x^5 + 3x^2 - 4}{e^{3x}}$

    Solution. 0; e^x grows faster than any polynomial

    (b) $\displaystyle\lim_{x\to 0^+} (1 - x)^{1/x}$

    Solution. $\dfrac{1}{e}$

    (c) $\displaystyle\lim_{x\to\infty} 2x - \sqrt{4x^2 - 5x}$

    Solution. $\dfrac{5}{4}$

11. (8 pts) A kite is flying at an angle of elevation of 45 degrees. The kite string is being taken in at the rate of 8 feet per minute. If the angle of elevation does not change, how fast is the kite losing altitude?

    Solution. The altitude h and length l are related by $\sin 45 = \frac{h}{l}$. Taking derivates we find $\dfrac{dh}{dt} = 4\sqrt{2}$.

FIGURE 1. Problem 12

12. (8 pts) The strength of a wooden beam is proportional to the product of its width W and the square of its depth D (see Figure 1). What is the width W and the depth D of the strongest beam that can be cut from a cylindrical log that is 2 feet in diameter?

    Solution. $W = \frac{2}{\sqrt{3}}$. $D = \sqrt{\frac{8}{3}}$.

13. (8 pts) Joe, the owner of Big Joe's Donuts, finds that at a price of \$5 per dozen he sells 12 dozen donuts every hour. He has found that if he lowers the price to 3 dollars a dozen, then he can sell 20 dozen donuts every hour. Assuming the demand curve is linear, what price should Joe charge to maximize his revenue?

    Solution. \$4

14. (12 pts) Let $f(x) = \dfrac{x}{(x+3)^2}$. Give the following information, and sketch the graph of f.

Vertical asymptotes	__________
End behavior	__________
Intercepts	__________
Local maxima	__________
Local minima	__________
Inflection points	__________
Intervals on which $f(x)$ is:	
Increasing	__________
Decreasing	__________
Concave up	__________
Concave down	__________

Solution. Vertical asymptote at $x = -3$.
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 0$.
x-intercept: $(0, 0)$, y-intercept: $(0, 0)$.
Local maximum: $(3, 1/12)$.
Local minima: None.
Inflection point: $(6, 2/27)$.
Increasing: $(-3, 3)$.
Decreasing: $(-\infty, -3) \cup (3, \infty)$.
Concave up: $(6, \infty)$.
Concave down: $(-\infty, -3) \cup (-3, 6)$.
The graph is shown in Figure 2.

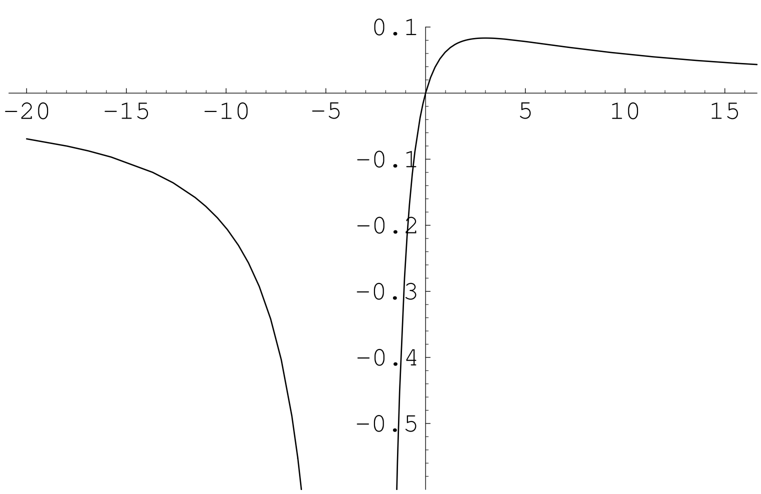

FIGURE 2. Problem 14