MATH 112, SECTION 9, WINTER 2003
TEST 4, MARCH 19-20
NO CALCULATORS.

INSTRUCTOR: JASON GROUT

Abstract. Fourth test covering sections 4.1-4.3.

Part I. Multiple Choice

Instructions: Circle the best answer.

1. (5 pts) Which of the following is an antiderivative of $\dfrac{1}{3-x}$?
   (a) $\ln|3-x|$
   (b) $\ln|3-x|+5$
   (c) Both of the above.
   (d) None of the above.
   Solution. (d). Both of the first answers are missing the negative sign on the ln.
2. (5 pts) Let C be a constant. The general antiderivative of $\dfrac{1}{\sqrt{1-x^2}}$ is
   (a) $\tan^{-1}x+C$
   (b) $\sec^{-1}x+C$
   (c) $\sin^{-1}x+C$
   (d) None of the above.
   Solution. (c).
3. (5 pts) Which of the following are solutions to the differential equation
   $$\frac{d^2s}{dt^2} = -\omega^2 s$$
   (a) $s=3\cos(\omega t)$
   (b) $s=5\sin(\omega t)$
   (c) $s=3\cos(\omega t)+5\sin(\omega t)$
   (d) All of the above.
   (e) None of the above.
   Solution. (d).

Part II. Short Answer

Instructions: Show your work. Clearly mark your answers. No calculators.

1. (5 pts each) Find the general antiderivatives of the following functions. Let k be a constant.
   (a) e^{-kx} ***Solution.*** $-\dfrac{ke^{-kx}}{k}+C$
   (b) $\sin kx$ ***Solution.*** $-\frac{\cos kx}{k}+C$
   (c) $k\cosh x$ ***Solution.*** $k\sinh x+C$
   (d) $3x^{-2}+5x^{-1}+7x^2$ ***Solution.*** $-3x^{-1}+5\ln x+\frac{7x^3}{3}+C$
2. (8 pts each) Solve the following differential equations, given the boundary conditions. Assume that y is a function of x.
   (a) $2y=\dfrac{\csc^2 x}{y'}$, $y(\frac{\pi}{2})=1$. ***Solution.*** $y=\sqrt{-\cot x+1}$
   (b) $y'+1=-y'x^2$, $y(1)=\frac{\pi}{4}$ ***Solution.*** $y=-\arctan x+\frac{\pi}{2}$
   (c) $y'-3=-4y$, $y(0)=0$ ***Solution.*** $y=\dfrac{3e^{-4x}-3}{-4}$
3. (5 pts) Prove that $\dfrac{a^{bx}}{b\ln a}+C$ is the general antiderivative of a^{bx}. ***Solution.*** The derivative of $\frac{a^{bx}}{b\ln a}+C$ is $\frac{a^{bx}(\ln a)(b)}{b\ln a}=a^{bx}$. Therefore, the antiderivative of a^{bx} is as stated.
4. (5 pts) A projectile is thrown downward from the top of a building at a speed of 3 ft/s. If the projectile hits the ground 2 seconds later, how high is the building? Assume the only acceleration acting on the projectile is $g=32\,\text{ft/s}^2$, the constant of gravity. ***Solution.*** 70 ft
5. (10 pts) A mass Q of a radioactive substance decays at a rate proportional to the mass of the substance.
   (a) Set up a differential equation describing the change in mass of the substance with respect to time. ***Solution.*** $\dfrac{dQ}{dt}=kQ$, where k is a constant.
   (b) A certain lump of the substance decays from 27 grams to 9 grams in 1 day. Solve the differential equation modeling the situation and find when there will be only 1 gram remaining. ***Solution.*** At the end of 3 days.
6. (10 pts) A certain plant grows in height at a rate inversely proportional to the square root of its height.
   (a) Set up a differential equation describing the change in height with respect to time. ***Solution.*** $\dfrac{dH}{dt}=\frac{k}{\sqrt{H}}$, where k is a constant.
   (b) It takes 8 years for the plant to reach a height of 4 meters. Solve the differential equation and find how tall the plant will be when it is 27 years old. ***Solution.*** 9 meters.
7. (10 pts) The temperature Q of a cup of water is found to change with respect to time at a rate proportional to the difference between its temperature and A, the temperature of the surrounding air.

(a) Set up a differential equation describing the rate of change of temperature of the cup of water with respect to time. ***Solution.*** $\frac{dQ}{dt} = k(Q - A)$, where k is a constant.

(b) Jason places a cup of water with temperature 10 degrees Celsius in a freezer that is kept at -10 degrees Celsius. One hour later, he notes that the temperature of the cup is 5 degrees Celsius. Solve the differential equation modeling the situation to find a function $Q(t)$ that gives the temperature of the cup at the time t. Prove your solution is correct. ***Solution.*** $Q(t) = -10 + 20e^{\ln(3/4)t}$. Using this, we see that the cup freezes at $t = \frac{\ln(1/2)}{\ln(3/4)}$ hours.