Exam 3 — Math 112 — Fall 2004

Write the best answer to each question in the box provided. For multiple choice and true/false questions, put your answer in the blank next to the question number.

1. (5 pts) T/F The global maximum of $e^{-x^2}$ is 0.

2. (5 pts) MC Suppose $f$ is twice differentiable everywhere. Which of the following conditions guarantees the existence of a local maximum at $x = 0$?

   (a) $f(0) = 0, f''(0) = 1.$
   (b) $f(0) = 0, f''(0) = -1.$
   (c) $f'(0) = 0, f''(0) = 1.$
   (d) $f'(0) = 0, f''(0) = -1.$
   (e) None of the above.

3. (5 pts) MC The global minimum of $3x - x^3$ for $x \in [-2, 3]$ occurs at $x =$

   (a) -2
   (b) -1
   (c) 0
   (d) 1
   (e) 2
   (f) None of the above

4. (5 pts) A cube’s side length changes from 1 inch to 1.1 inches. Using a linear approximation of the formula $V = s^3$, what is the approximate change in volume?

   (a) 1.1 inches
   (b) 1.21 inches
   (c) 1.3 inches
   (d) 1.1 inches
   (e) .21 inches
   (f) .3 inches

5. (5 pts) MC Newton’s method is used to find a zero of $x^2 - 9$. If $x_1 = 2$, find $x_2$.

   (a) 3
   (b) $-\frac{3}{2}$
   (c) $\frac{5}{2}$
   (d) 4
   (e) $\frac{3}{2}$
   (f) 0
   (g) None of the above.

6. (5 pts) MC If $f''(x) = x(x - 1)^2(x - 2)^3$, the points of inflection of $f$ are

   (a) 0
   (b) 1
   (c) 2
   (d) -1
   (e) 0,1
   (f) 0,2
   (g) 1,2
   (h) 0,1,2
   (i) 0,1,-1
   (j) 0,2,-1
   (k) 1,2,-1
   (l) 0,1,2,-1
7. (6 pts) If the first derivative of $f$ is $f'(x) = e^{-6x}(x^2 - 4)(x^2 + 1)x$, give all the intervals on which $f$ is

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8. (6 pts) If the second derivative of $f$ is $f''(x) = \frac{\ln |x|}{\tan^{-1}x}$, state the intervals on which $f$ is

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9. (6 pts) Suppose Frank drives 2 miles to school in exactly 3 minutes.

If the posted speed is 35 miles per hour, did Frank exceed the speed limit?

If so, by how much does the Mean Value Theorem say Frank was speeding?

If not, explain your conclusion:

10. (6 pts) Approximate $\sqrt{66}$ using differentials (linear approximation).
11. (6 pts) Find all the critical points of the function \( f(x) = e^{\sin x + \cos x} \).

12. (10 pts) Find the \( x \)-values of all local extrema of \( f(x) = x + \frac{9}{x} \) on the interval \([1, 18] \). What is the absolute maximum? What is the absolute minimum. Use the first derivative test to justify your work.

   x-values:
   Maximum:
   Minimum:

13. (10 pts) Find the point on the curve \( y = \sqrt{64 - 6x} \) that is closest to the origin \((0, 0)\).
This portion of the exam is take home. You must justify all your work. You may ONLY use your calculus textbook, your notes, and a calculator (you should not need a calculator). For instance, you are not allowed to communicate with anyone about this problem, orally or through email. You may not use any outside resources. This is due by the beginning of class Thursday morning.

1. (10 pts) An open rectangular box is to be made from a 3 by 8 foot piece of cardboard by cutting a square out of each corner and then folding up the sides and taping together at the corners. What is the length of the side of the square that should be cut out to maximize the volume?

2. (10 pts) Jed is building a shack in downtown Hollywood for his offices. The floor of the shack needs to be a square. He plans to make the shack with a dirt floor (i.e., he is just building the 4 walls and the ceiling). The plywood he is using costs $25 per square foot, and the land he wants to buy for the shack costs $75 per square foot. He needs the shack to enclose 2000 cubic feet. Find the dimensions of the shack that will minimize his cost.