Writing Assignment 10 - Theorems

There are some key theorems in the book that you should know. They have specific names, which you should be familiar with. The following statements are rough descriptions of these theorems. For each statement:

(a) Write the name of the theorem it describes,
(b) write a formal statement of the theorem (the hypothesis and conclusion),
(c) and if applicable, draw a representative picture of the theorem.

1. Average speed equals instant speed at a point.

2. You can’t get from the highest point to the lowest point without passing through every point between.

3. At some point, instant value is equal to average value.

4. On a closed interval, a continuous function always has a maximum and a minimum.

5. If you pass through zero twice, your slope is zero somewhere between.

6. The derivative of a composition is the derivative of the outside, evaluated at the inside, multiplied by the derivative of the inside.

7. If a function is concave up at a critical point, then the function has a minimum value at that point. You get a maximum value when the function is concave down.

8. If a function changes from increasing to decreasing at a point, then it has a maximum value at that point. Similarly, if the function changes from decreasing to increasing, you have found a local minimum.

Answer the following questions.

1. If $F$ and $G$ are two antiderivatives of $f$, then what can be said about $F - G$?

2. When is it true that $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$? What should be true of $f$ and $g$ to permit passing a limit inside $f$?

3. Suppose $A(x) = \int_{a}^{x} f(t) \, dt$. When $f$ is continuous, what is $A'(x)$? What is $\frac{d}{dx}[A(g(x))]$? Calculate $\frac{d}{dx} \int_{\sqrt{x}}^{x^3} \sin(t^2) \, dt$?