Notice: Read the questions carefully. Put your answers in the provided boxes. In order to receive full credit, you will need to neatly show your work on these pages and simplify your answers appropriately. Do not attach extra pages. If the provided space is insufficient, use the blank sides of adjacent pages.

1. (6 pts) If $f$ is twice differentiable in the interval $(1, 3)$, $f'(2) = 0$, and $f''(2) = -3$, then which of the following is always true? Circle your answer.

   (a) $f$ has a local minimum at $x = 2$.
   (b) $f$ has a local maximum at $x = 2$.
   (c) $f$ has an inflection point at $x = 2$.
   (d) $f$ has both an inflection point and a local minimum at $x = 2$.
   (e) $f$ has both an inflection point and a local maximum at $x = 2$.
   (f) None of the above is always true.

2. (6 pts) If $g$ is twice differentiable in the interval $(4, 6)$, $g'(5) = 0$, and $g''(5) = 0$, then which of the following is always true? Circle your answer.

   (a) $g$ has a local minimum at $x = 5$.
   (b) $g$ has a local maximum at $x = 5$.
   (c) $g$ has an inflection point at $x = 5$.
   (d) $g$ has both an inflection point and a local minimum at $x = 5$.
   (e) $g$ has both an inflection point and a local maximum at $x = 5$.
   (f) None of the above is always true.

The 2nd derivative test is inconclusive if the 2nd derivative is 0.

Though $g''(5) = 0$, doesn't mean that there is an inflection point at $x = 5$. See Problem 4 for a counterexample.
3. (16 pts) If the first derivative of $f$ is $f'(x) = x e^x (x^2 - 9)$, give all the intervals on which $f$ is increasing or decreasing. Also find the locations of the relative maxima and minima. Assume that $f$ is defined everywhere.

$$f'(x) = 0 \Rightarrow x = 0, -3, 3$$

Increasing: $(3, \infty) \cup (-3, 0)$
Decreasing: $(-3, 0) \cup (0, 3)$
Relative maxima at $x = 0$
Relative minima at $x = -3, 3$

(a) $f$ not defined - nowhere
(b) $f'$ not defined - nowhere
(c) $f'' = 0$ - $x = 0, \pm 3$.

4. (12 pts) If the second derivative of $g$ is $g''(x) = \frac{(x^2 - 4)(x - 3)^2}{x}$, give all the intervals on which $g$ is concave up or concave down. Also, find the locations of inflection points of $g$. Assume that $g$ is defined everywhere.

$$g'' = 0 \Rightarrow x = -2, 0, 3$$

Concave up: $(-2, 0) \cup (2, 3) \cup (3, \infty)$
Concave down: $(-\infty, -2) \cup (0, 2)$
Inflection points at $x = -2, 0, 2$
5. (10 pts) Find the absolute maximum and minimum points of \( f(x) = x^3 - 3x + 1 \) on the interval \([0, 3]\). Give both the \(x\) and \(y\) coordinates of the maximum and minimum.

\[
f'(x) = 3x^2 - 3 = 0
\]
\[
x^2 - 1 = 0
\]
\[
x = \pm 1
\]

-1 is not in \([0, 3]\), so we ignore it.

\[
\begin{array}{c|c|c}
 x & f(x) & \text{endpoints} \\
 0 & 1 & \\
 1 & -1 & \\
 3 & 27 - 9 + 1 = 19 & \end{array}
\]

Absolute max: \((3, 19)\)

Absolute min: \((1, -1)\)

6. (14 pts) Draw a differentiable function on the axes provided that clearly has the following properties. Put a small circle around inflection points and a small square around relative extrema.

(a) the function is concave up on \((-5, -2)\) and on \((1, 5)\);
(b) the function is concave down on \((-2, 1)\);
(c) the function is increasing on \((-5, 0)\);
(d) the function is decreasing on \((0, 5)\).
7. (18 pts) In a local orchard, if only one tree is planted, then the tree produces 49 baskets of fruit. For each additional tree planted, each tree produces one basket less (so if 2 trees are planted, each tree produces 48 baskets of fruit). The orchard only has space for up to 40 trees. How many trees should be planted to give the most total fruit? Make sure to justify why your answer is a maximum or minimum.

\[
\text{Total Fruit} = (\text{fruit per tree})(\text{# of trees})
\]
\[
= (50-x)(x) = 50x - x^2
\]
\[
\frac{d(\text{Total Fruit})}{dx} = 50 - 2x = 0
\]
\[
x = 25 \text{ trees is the location of a critical point}
\]
\[
\text{Check endpoints and critical points:}
\]
\[
\begin{array}{c|c}
\text{critical points} & \text{critical points}
\end{array}
\]
\[
\begin{array}{c|c|c}
\frac{\text{Total Fruit}}{x} & \text{closed interval} & \text{we checked the critical points and endpoints.}
\end{array}
\]

8. (18 pts) You are blowing up a spherical balloon by pumping in air so that the radius is expanding at 2 feet per minute. You want to make sure that you don’t stretch the material of the balloon too quickly. How fast is the surface area of the balloon increasing when the radius of the balloon is 3 feet? (Hint: The volume of a sphere is \(\frac{4}{3}\pi r^3\) and the surface area is \(4\pi r^2\), where \(r\) is the radius). Give your answer with the correct units.

\[
\frac{dS}{dt} = 4\pi (2r) \frac{dr}{dt} = 8\pi r \frac{dr}{dt}
\]
\[
r = 3, \frac{dr}{dt} = 2, \text{ so } \frac{dS}{dt} = 8(3)(2)\pi = 48\pi \text{ ft}^2\text{ /min}
\]

There are 100 total points.