Test 4 Study Guide

To review for Test 4, you should know the things mentioned in the chapter summary and key terms for sections 9.1–9.4 (pp. 558–559) and sections 10.1 and 10.3 (p. 606).

You should also know how to work problems involving the following concepts. For (almost) every item, there are a number of practice problems suggested which will help you review the item. Unless noted, the problem numbers refer to problems in the Chapter Review sections at the end of the chapter.

Chapter 9

1. Know how to evaluate multivariable functions at specific points (see 4–7).

2. Graphing multivariable functions. You should know how to graph the first octant portion of a plane (see 8–13). You should be able to analyze a function, slice it with various planes (i.e. compute the traces of the function), and choose its graph from a number of graphs (see p. 506, 21–26).

3. Partial derivatives. You should know the various notations for partial derivatives of various orders (remember that the order reverses when using the two notations)

   \[ f_{xy} = \frac{\partial f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} f \right). \]

   You should know how to evaluate partial derivatives at specific points and understand what the partial derivative means graphically. See problems 14–31.

4. Finding relative extrema and saddle points. You should know how to find locations of relative minima, relative maxima, and saddle points of a multivariable function. You should also know when you can’t say anything about a point (when the test is inconclusive). See problems 32–40.

5. Lagrange multipliers. You should be able to find possible candidates for extrema of a multivariable function with a given constraint function using Lagrange multipliers. See problems 41–44. You should be able to do this even when the function has more than two variables (see p. 539, 9–10, and also example 3 on p. 537). (Hint: If you have 4 variables \( w, x, y, \) and \( z \) and you want to minimize \( f(w, x, y, z) \) subject to the constraint \( g(w, x, y, z) = 0 \), then just construct the function \( F(w, x, y, z, \lambda) = f(w, x, y, z) - \lambda g(w, x, y, z) \). Take the partial derivatives of \( F \) with respect to each variable and set them equal to zero: \( F_w = 0, F_x = 0, F_y = 0, F_z = 0, \) and \( F_\lambda = 0 \). Then solve that system to find points \( (w, x, y, z) \).) Remember that the points that you get out of the Lagrange multiplier method are only possible extrema. Also, remember that you cannot use the tests from 9.3 to test these points—usually the points you get out of the Lagrange multiplier method are not critical points of your function.

Chapter 10

1. Setting up differential equations. You should know how to set up a differential equation if you are given that a rate is equal or proportional or inversely proportional to several
things. For example, if the height of a pile of sand is changing at a rate that is proportional to the cube root of one plus the height squared and inversely proportional to the height, then we have the differential equation
\[
\frac{dh}{dt} = k \left( \sqrt[3]{1 + h^2} \right) \left( \frac{1}{h} \right),
\]
where \( k \) is a constant. See problems 44, 48, 50, and the quiz problem.

2. Solving separable differential equations. You should be able to solve a separable differential equation and give the general solution (the solution with the constant). If it is an initial value problem (i.e., you are given an initial condition), then you should be able to get the particular solution. You should be able to correctly work out the details when you have a solution that involves absolute values (i.e., exponential growth or decay). See problems 13–23, 25–27, 29–31, 33–35, 43–48, 50–52, 57.

3. Euler’s method. You should be able to approximate the solution of a differential equation using Euler’s method. See problems 38–41.