Math 302 Outcome Statements with Lectures Outlines

1 Introduction

Outcomes:

A. Discuss how multivariable calculus and algebra are related to engineering and science.

Lecture:

1. Have students recall applications learned in single variable calculus such as:
   (a) instantaneous rates
   (b) optimization
   (c) lines tangent to a planar curve
   (d) area bounded between curves
   (e) relations between distance travelled, speed and acceleration for objects moving along a strait line
   (f) arclength, volume of revolution, surface areas of revolution
   (g) mass, center of mass and centroid for a flat plate
2. Discuss applications of multivariable calculus and linear algebra:
   (a) optimization on surfaces
   (b) representations of surfaces
   (c) tangent planes to surfaces
   (d) curvilinear motion
   (e) surface area, mass, center of mass and centroid for a curved plate
   (f) volumes, mass, center of mass and centroid for solids
   (g) paths of heat seeking or heat repelling objects
   (h) calculations of flux through surfaces
   (i) representations in high dimensions
   (j) solutions to large systems of linear equations
   (k) representations of linear transformations
   (l) numerical methods for solving linear systems
3. Discuss how the different mathematical topics are integrated in engineering and science applications:
   (a) Partial Differential Equations
   (b) Ordinary Differential Equations
   (c) Multivariable Calculus
   (d) Linear Algebra

2 Vectors in Two and Three Dimensions

A. Evaluate the distance between two points in 3-space.
B. Define vector and identify examples of vectors.
C. Be able to represent a vector in each of the following ways for $n = 2, 3$:
   (a) as a directed arrow in n-space
   (b) as an ordered n-tuple
   (c) as a linear combinations of unit coordinate vectors
D. Carry out the vector operations:
   (a) addition
   (b) scalar multiplication
   (c) magnitude (or norm or length)
   (d) normalize a vector (find the vector of unit length in the direction of a given vector)
E. Represent the operations of vector addition, scalar multiplication and norm geometrically.
F. Recall, apply and verify the basic properties of vector addition, scalar multiplication and norm.
G. Model and solve application problems using vectors.

Reading: Multivariable Calculus 1.1, Linear Algebra 1.1

Homework: MC 1.1: 1ac,2,4d,7,9ace,12df,13bg,14b,15b,17bc LA 1.1: 1c,3c,4c,5c,6,14

Outcome Mapping:

A. 1-6 (LA 1.2: 13-16)
B. A1,A2,7
C. 7,12,13,14 (LA 1.1 1,13)
D. 10,15 (LA 1.1 7-10,11-12)
E. 8,10 (LA 14,19,22)
F. 11,16-21,A3,A4 (LA 1.1 18,23,24)
G. A5,6,7

Lecture:

1. Definition of a vector.
   (a) Present examples of scalar quantities and vector quantities.
      i. Scalar quantities- distance, speed, mass, length, temperature, voltage, heat.
      ii. Vector quantities - displacement, velocity, momentum, force.
   (b) Define a vector quantity as a scalar quantity together with a direction and relate examples (e.g., displacement is a distance in a certain direction).
   (c) Define the length or norm of a vector as the scalar quantity associated with it.
   (d) Define the zero vector.
   (e) Define equality for vectors.
2. Representations of vectors in 2- and 3-space.
   (a) Illustrate representations of vectors as an arrow in 2- and 3-space.
   (b) Identify vectors that are equivalent. Emphasize that position is not a part of the definition of a vector.
   (c) Represent a vector in $\mathbb{R}^n$ using components.
   (d) Calculate the length of a vector.
   (e) Translate a vector to its equivalent position vector.
   (f) Define the one-to-one correspondence between the vectors in n-space and the points of n-space. Work examples.
3. Vector addition and scalar multiplication
   (a) Define vector addition and scalar multiplication component-wise.
   (b) Illustrate vector addition and scalar multiplication geometrically.
   (c) Define the resultant as the sum of vectors.

(d) Define the basic properties of vector addition: commutativity, associativity, additive identity, additive inverse. Illustrate these properties geometrically and algebraically.

(e) Define the basic properties of scalar multiplication: distributive laws, associativity, multiplicative identity, multiplication by 0 and -1. Illustrate these properties geometrically and algebraically.

4. Basic unit vector

   (a) Define the unit vector **i**, **j**, and **k**.

   (b) Convert an ordered triple to unit vector notation.

   (c) Sketch a vector given in unit vector notation.

   (d) Normalizing a vector

   (e) Define a unit vector in general.

   (f) Find a unit vector in a given direction.

5. Applications

   (a) Use vectors to demonstrate at least one of the following:

      i. The diagonals of a parallelogram bisect each other.

      ii. The midpoints of the sides of a quadrilateral are the vertices of a parallelogram.

   (b) Illustrate how to construct and analyze force diagrams using vectors.

3 Vector Products

A. Evaluate a dot product from the angle formula or the coordinate formula.
B. Interpret the dot product geometrically.
C. Evaluate the following using the dot product:

   i. the angle between two vectors.
   ii. the magnitude of a vector.
   iii. the projection of a vector onto another vector.
   iv. the component of a vector in the direction of another vector.
   v. the work done by a constant force on an object.

D. Evaluate a cross product from the angle formula or the coordinate formula.
E. Interpret the cross product geometrically.
F. Evaluate the following using the cross product:

   i. the area of a parallelogram.
   ii. the area or a triangle.
   iii. physical quantities such as moment of force and angular velocity.

G. Find the volume of a parallelepiped using the scalar triple product.
H. Recall, apply and derive the algebraic properties of the dot and cross products.

Reading: Multivariable Calculus 1.2-3, Linear Algebra 1.2

Homework: MC 1.2: 2adegj,3,7be,8a,13 LA 1.2:17,18,19,32,34,40; MC 1.2: 2knop,4,5,6,7dfgh,8d,10,16,17,18

Outcome Mapping:

A. 1,2,7 (LA 1.2: 1-6,17)
B. 3,9,19 (LA 1.2: 40,41)
C. 2,13,14,15 (LA 1.2: 7-12,18-29,30-32,34-39,42-45)
D. 1,2,7
E. 9,16
F. 5
G. 6,8
H. 4,10,17,18,20 (LA 1.2: 46-68)

1. Introduce the dot product as a product of vectors with scalar output. Contrast with scalar multiplication.
2. Motivate the dot product as an operation that can be used to find the component of a vector in a certain direction, angles between lines and planes, and the work done by a force.
3. Define dot product
   (a) in terms of the angle between two vectors.
   (b) in terms of the coordinate formula.
4. Give an example of calculating the dot product in both ways.
5. Use the definition of dot product to interpret the dot product as the length of the projection of one vector onto another.
6. Prove that the coordinate formula is equivalent to the angle formula in 2-space.
7. Define orthogonality and relate it to dot product.
8. Properties of inner product.
   (a) State the general properties: linearity, symmetry, positive-definiteness, distributivity, the triangle inequality, the parallelogram inequality.
   (b) Prove the parallelogram inequality.
9. Work examples of the following applications:
   (a) Work done by a force.
   (b) Finding the component of projection of one vector onto another.
   (c) Finding the angle between two lines.
   (d) Finding the distance between a point and a line

Lecture B:

1. Introduce the cross product as a product of vectors in 3-space with vector output. Contrast with the dot product.

2. Motivate the cross product as an operation that can be used to find orthogonal vectors, moments of force, areas of triangles and parallelograms, and volumes of parallelepipeds.
3. Define cross product.
   (a) in terms of the angle between two vectors and the right-hand rule.
   (b) in terms of the coordinate formula (use determinants).
4. Give an example of calculating the cross product in both ways.
5. Use the definition of cross product to interpret the cross product as the vector with magnitude equal to the area of the parallelogram formed by the two given vectors and direction perpendicular to the two given vectors.
6. Prove that the coordinate formula is equivalent to the angle formula in 2-space.
7. Determine the cross product of the unit vectors.
8. Properties of the cross product.
   (a) State the general properties: scalar multiplication, distributivity, anticommutativity.
   (b) Show that the cross product is not associative using unit vector.
   (c) Use the properties to evaluate the cross product of two vectors represented as the linear combination of unit vectors.
9. Work examples of the following applications:
   (a) area of a triangle.
   (b) volume of a parallelepiped.
   (c) moment of force.
   (d) angular velocity.

4 Planes in Space

A. Find the equation of a plane in 3-space given a point and a normal vector, three points, a sketch of a plane or a geometric description of the plane.
B. Determine a normal vector and the intercepts of a given plane.
C. Sketch the graph of a plane given its equation.
D. Determine the angle between two planes.

Reading: Multivariable Calculus 1.3, Linear Algebra 1.3

Homework: MC 1.3: 1bejln,2b,3abc,4abc LA 1.3: 25

Outcome Mapping:

A. 1,3,5 (LA 1.3: 7-10,24,25)
B. 2,4 (LA 1.3: 18,19)
C. 4
D. 2 (LA 1.3: 43,44)

Lecture:

1. The equation of the plane
   (a) Give examples of equations of planes parallel to the coordinate planes or parallel to a coordinate axis.
   (b) Derive the equation of a plane in vector form.
   (c) Translate the vector form the equation of a plane to scalar form.
2. Work examples of determining the equation of a plane given:
   (a) three points in the plane
   (b) two vectors in the plane
   (c) one point and a parallel plane
   (d) the graph of a plane
3. Show how to graph a plane in the case of
   (a) three intercept points with the coordinate axes.
   (b) two intercept points with the coordinate axes.
   (c) one intercept points with the coordinate axes.
4. Angle between planes
   (a) Define what is meant by the angle between two planes.
   (b) Devise a strategy for finding the angle between two planes.
   (c) Apply the strategy to an example.

5 Lines in Space

A. Represent a line in 3-space by a vector parameterization, a set of scalar parametric equations or using symmetric form.
B. Find a parameterization of a line given information about
   (a) a point of the line and the direction of the line or
   (b) two points contained in the line.
   (c) the direction cosines of the line.
C. Determine the direction of a line given its parameterization.
D. Find the angle between two lines.
E. Determine a point of intersection between a line and a surface.
F. Find the equation of a plane determined by lines.

Reading: Multivariable Calculus 1.5, Linear Algebra 1.3

Homework: MC 1.5: 1ac,2bd,3bfimps,4bde,8,10,16ab,20

Outcome Mapping:

A. 3,4 (LA 1.3: 1-6,16)
B. 3,4,5,6,7,8,9,12 (LA 1.3: 1-6,11-15,20-23)
C. 1,12 (LA 1.3: 17,23)
D. 2
E. 10,11,14 (LA 1.3: 45,46)
F. 9

Lecture:

1. Discuss possible generalizations of the representation of a line in 2-space to the representation of a line and plane in 3-space.
2. Derivation
   (a) Derive the vector form of the equation of a line.
   (b) Convert the vector form to scalar parametric form and symmetric form.
3. Examples
   (a) Determine the direction of a line given its parameterization.
   (b) Work examples of determining the equations of a line given
      i. a point and a direction.
      ii. two points.
      iii. a point and a parallel line
   (c) Find the angle between two lines.
   (d) Determine a point of intersection between a line and a surface.
   (e) Find the equation of a plane determined by lines.
4. Direction cosines
   (a) Define the direction cosines.
   (b) Given a line, determine its direction cosines.
   (c) Given the direction cosines of a line and a point, find the equation of the line.

6 Systems of Linear Equations

Outcomes:

A. Define linear equation and system of linear equations. Define solution and solution set for both an linear equation and a system of linear equations.
B. Relate the following types of solution sets of a system of two or three variables to the intersections of lines in a plane or the intersection of planes in three space:
   i. a unique solution.
   ii. infinitely many solutions.
   iii. no solution.
C. Represent a linear system as an augmented matrix and vice versa.
D. Transform a system to a triangular pattern and then apply back substitution to solve the linear system.
E. Represent the solution set to a linear system using parametric equations.

Reading: Linear Algebra 2.1

Homework: 2.1: 1,4,5,8,10,12,13,15,17,21,24,25,28,29,31,37,39

Outcome Mapping:

A. 1-6, 7-10
B. 15-18
C. 27-30, 31-32
D. 19-24, 25-26, 33-38
E. 11-14, 39-40

Lecture:

7 Direct Methods for Solving Linear Systems

Outcomes:

A. Identify matrices that are in row echelon form and reduced row echelon form.
B. Determine whether a system of linear equations has no solution, a unique solution or an infinite number of solutions from its echelon form.
C. Apply elementary row operations to transform systems of linear equations.
D. Solve systems of linear equations using Gaussian elimination.
E. Solve systems of linear equations using Gauss-Jordan elimination.
F. Define and evaluate the rank of a matrix.
G. Apply the Rank Theorem relate the rank of an augmented matrix to the solution set of a system in the case of homogeneous and nonhomogeneous systems.
H. Model and solve application problems using linear systems.

Reading: Linear Algebra 2.2

Homework: 2.2: 1,2,3,4,5,10,12,13,19,21,23,26,28,30,35,37,39,40,48; 2.4: 16

Outcome Mapping:

A. 1-8,24
B. 39-44
C. 9-14,15-16,17-18,19-22
D. 25-34
E. 23
F. 35-38
G. 45-52, (2.4: 1-47)

Lecture:

8 Spanning Sets and Linear Independence

Outcomes:

A. Explain what is meant by the span of a set of vectors both geometrically and algebraically.
B. Determine the span of a set of vectors. Determine if a given vector is in the span of a set of vectors.
C. Define linear independence.
D. Determine whether a set of vectors is linearly dependent or linearly independent. For sets that are linearly dependent, determine a dependence relation.
E. Prove theorems about span and linear independence.

Reading: Linear Algebra 2.3

Homework: 2.3: 2,4,7,11,14,15,24,26,34,36,44,46 2.4:19

Supplemental Problems:

A1. Study the definition of linear independence. Write it from memory.

Outcome Mapping:

A. 13-16,17
B. 1-6,7-12
C. A1
D. 22-31
E. 18-21,42-48

Lecture:

9 Matrix Operations

Outcomes:

A. Perform the operations of matrix addition, scalar multiplication, transposition, and matrix multiplication.
B. Represent matrices in terms of double subscript notation.
C. Recall and demonstrate that the cancellation laws for scalar multiplication do not hold for matrix multiplication.
D. Use matrices and matrix operations to model application problems.
E. Represent matrix multiplication in terms of blocks.

Reading: Linear Algebra 3.1

Homework: 3.1: 2,4,6,8,16,17,18,19,22,32,36,39

Outcome Mapping:

A. 1-16,35-38
B. 39-40
C. 17-18
D. 19-20,21-22
E. 23-28,29-30,31-34,41

Lecture:

10 Matrix Algebra

Outcomes:

A. Recall and apply the algebraic properties for matrix addition, scalar multiplication, matrix multiplication, and transposition.
B. Prove algebraic properties for matrices.
C. Apply the concepts of span and linear independence to matrices.
D. Recall that matrix multiplication is not commutative in general. Determine conditions under which matrices do commute.

Reading: Linear Algebra 3.2

Homework: 3.2: 2,4,6,9,14,22,24,36a,37,39,42,44

Outcome Mapping:

A. 1-4
B. 17-22,29-36,37-43,44-47
C. 5-8,9-12,13-16
D. 23-28

Lecture:

11 The Inverse of a Matrix

Outcomes:

A. Define the inverse of a matrix.
B. Recall the Fundamental Theorem of Invertible Matrices and the properties of invertible matrices. Prove theorems involving matrix inverses.
C. Recall and apply the formula for the inverse of 2×2 matrices.
D. Demonstrate the relationship between elementary matrices and row operations. Determine inverses of elementary matrices.
E. Compute the inverse of a matrix using the Gauss-Jordan method.
F. Solve a linear equation using matrix inverses.

Reading: Linear Algebra 3.3

Homework: 3.3: 2,10,11,17a,20,22,24,26,28,34,36,38,42,43,44,48,50,52

Supplemental Problems:

B1. Study the definition of matrix inverse. Write it from memory.

Outcome Mapping:

A. B1
B. 14-19,41-47
C. 1-10
D. 24-30,31-38,39-40
E. 48-63
F. 11-13,20-23

Lecture:

12 The LU Factorization

Outcomes:

A. Find the LU factorization of a matrix.
B. Use the LU factorization of a matrix to solve a system of linear equations.
*C. Find the $P^T LU$ factorization of a matrix.
*D. Use that $P^T LU$ factorization of a matrix to solve a system of linear equations.
*E. Find the inverse of a matrix using the LU factorization.

Reading: Linear Algebra 3.4

Homework: 3.4: 4,6,8,10,12,13,14

Outcome Mapping:

A. 7-12,13-14
B. 1-6
*C. 19-22,23-25
*D. 27-28
*E. 15-18,30

Lecture:

13 Subspaces, Basis, Dimension, and Rank

Outcomes:

A. Define subspace of $\mathbb{R}^n$. Determine whether or not a given set of vectors forms a subspace of $\mathbb{R}^n$.
B. Define row space, column space, and null space for a matrix. Determine whether or not a given vector is in one of these spaces.
C. Define basis and dimension. Given a subspace, determine its dimension and a basis. Verify whether or not a given set of vectors is a basis for the subspace.
D. Define rank and nullity. Determine the rank and nullity of a given matrix.
*E. Prove and recall theorems involving the rank, nullity, and invertibility of matrices.
*F. Find the coordinates of a vector with respect to a given basis.

Reading: Linear Algebra 3.5

Homework: 3.5: 1,2,3,4,8,11,12,15,17,19,28,30,35,36,40,42,46

Outcome Mapping:

A. 1-10
B. 11-16
C. 17-20,21-26,27-30,31-34,45-48
D. 35-42,43-44
*E. 55-64
*F. 49-50

Lecture:

14 Linear Transformations

Outcomes:

A. Define linear transformation. Determine whether or not a given transformation is linear.
B. Determine the matrix that represents a given linear transformation of vectors.
C. Prove and recall theorems involving linear transformations.
*D. Find compositions and inverses of linear transformations.

Reading: Linear Algebra 3.6

Homework: 3.6: 1,3,10,11,15,16,17,18,20,24,36,37,44

Outcome Mapping:

A. 1-10,46-51
B. 11-14,15-28
C. 29,40-45,52-55
*D. 30-35,36-39

Lecture:

15 Applications

Outcomes:

A. Model Markov processes. Determine the transition matrix, state vectors, probability vectors, and the steady state vectors.
*B. Model population growth using a Leslie matrix. Investigate the behavior of the growth.
*C. Relate adjacency matrices to graphs and digraphs. Investigate path lengths by taking powers of adjacency matrices.
*D. Solve problems involving error coding.

Reading: Linear Algebra 3.7

Homework: 3.7: 9,12,15,18,21,27,30,34,41,52

Outcome Mapping:

A. 1-18
B. 19-24
C. 25-60
*D. 61-75

16 Introduction to Eigenvalues and Eigenvectors

Outcomes:

A. Interpret the eigenvalue problem algebraically.

   i. Determine whether a given vector is an eigenvector.

   ii. Verify that a given value is an eigenvalue.

B. Interpret the eigenvalue problem geometrically. Determine eigenvalues and eigenvectors based on:

   i. an understanding of the linear transformation determined by the matrix

   ii. from the graph of the eigenspace.

C. Find the eigenvalues and eigenvectors of a general 2×2 matrix.

Reading: Linear Algebra 4.1

Homework: 4.1: 2,6,8,11,14,15,17,18,19,21,22,23,24,27,30

Outcome Mapping:

A. 1-6,7-12
B. 13-18,19-22
C. 23-26,27-30,31-34,35-38

Lecture:

17 Determinants

Outcomes:

A. Apply the Laplace Expansion to evaluate determinants of $n \times n$ matrices.
B. Recall and apply the properties of determinants to evaluate determinants, including:

   i. $\det(AB) = \det(A)\det(B)$
   ii. $\det(kA) = k^n \det(A)$
   iii. $\det(A^{-1}) = \dfrac{1}{\det(A)}$
   iv. $\det(A^T) = \det(A)$

C. Recall the effects that row operations have on the determinants of matrices. Relate to the determinants of elementary matrices.
D. Prove theorems involving determinants.
E. Evaluate matrix inverses using the adjoint method. Determine whether or not a matrix has an inverse based on its determinant.
F. Use Cramer's rule to solve a linear system.

Reading: Linear Algebra 4.2

Homework: 4.2: 5,8,13,17,24,26,27,30,32,37,39,40,46,48,49,51,53,54,60,61,64

Outcome Mapping:

A. 1-6,7-15,16-20
B. 35-38,47-52
C. 26-33,35-40
D. 21,41-44,53-56,66
E. 45-46,61-64,65
F. 57-60

Lecture:

18 Eigenvalues and Eigenvectors of $n \times n$ Matrices

Outcomes:

A. Given an $n \times n$ matrix, compute

   i. the characteristic polynomial

   ii. the eigenvalues

   iii. a basis for each eigenspace

   iv. the algebraic and geometric multiplicities of each eigenvalue

B. Solve application problems involving eigenvalues and eigenvectors.

C. Recall and prove theorems involving eigenvalues and eigenvectors.

Reading: Linear Algebra 4.3

Homework: 4.3: 4,5,9,15,16,19,20

Outcome Mapping:

A. 1-12

B. 15-22,26-31,33-38

C. 23-25,32,39-42

Lecture:

19 Similarity and Diagonalization

Outcomes:

A. Define similarity. Determine whether or not two matrices are similar.
B. Determine if a matrix is diagonalizable. Find the diagonalization of a matrix.
C. Find powers of a matrix using the diagonalization of a matrix.
D. Prove theorems involving the similarity and diagonalization of matrices.

Reading: Linear Algebra 4.4

Homework: 4.4: 1,3,6,8,11,15,16,40,41

Outcome Mapping:

A. 1-4,36-39
B. 5-7,8-15,24-29
C. 16-23
D. 30-35,40-50

Lecture:

20 Surfaces

A. Identify standard quadratic surfaces given their functions or graphs.
B. Sketch the graph of a quadratic surface by identifying the intercepts, traces, sections, symmetry and boundedness or unboundedness of the surface.

Reading: Multivariable Calculus 1.4

Homework: 1.4: 2,3bdgikm,4

Outcome Mapping:

A. 1,2,3
B. 3,4,6,7

Lecture:

1. Define quadratic surface.
2. Illustrate how to graph the ellipsoid and the hyperboloid of one sheet by identifying the intercepts, traces, sections, center, symmetry and boundedness or unboundedness of the surfaces.
3. Match basic quadratic equations to their surfaces. Justify the matching by analysis of the intercepts, traces, sections, symmetry and boundedness or unboundedness of the surfaces. Discuss the rationalization for the name of each.
   (a) the hyperboloid of two sheets
   (b) the elliptic cone
   (c) the elliptic paraboloid
   (d) the hyperbolic paraboloid
   (e) the parabolic cylinder
   (f) the hyperbolic cylinder
4. Given variations of the basic quadratic equations,
   (a) identify the surface represented.
   (b) sketch the surface by identifying intercepts, traces, sections, symmetry and boundedness or unboundedness of the surface.

21 Curves in Space

A. Identify the domain of a vector function.
B. Identify a curve given its parameterization.
C. Determine combinations of vector functions such as sums, vector products and scalar products.
D. Define limit, derivative and integral for vector functions.
E. Evaluate limits, derivatives and integrals of vector functions.
F. Find the line tangent to a curve at a given point.
G. Describe what is meant by arc length.
H. Evaluate the arc length of a curve.
I. Recall, derive and apply rules to combinations of vector functions for the following:
   (a) limits
   (b) differentiation
   (c) integration

Reading: Multivariable Calculus 1.6

Homework: 1.6: 1ac,2bd,3b,4c,5bc,6bd,8cd,13,14,C1,C2,C3,C4,C5

Outcome Mapping:

A. 1
B. 2
C. 3,4
D. C1
E. C2,C3,C4,13
F. 5,16
G. C5
H. 6
I. 7-12,14,15

Lecture:

1. Definitions
   (a) Relate a curve in 3-space to the path of an object moving in 3-space.
   (b) Define a curve in space as the graph of a vector function.
   (c) Define parameter and parameterization.
   (d) Define the domain of a function. Illustrate with an example.
   (e) Emphasize that for a given curve, the parameterization is ambiguous.
2. Model how to graph the following:
   (a) a straight line from parametric equations.
   (b) an ellipse.
   (c) a circular helix.
   (d) the twisted cubic $\mathbf{r}(t) = [t, t^2, t^3]$.
3. Find parameterizations for the following:
   (a) a shifted circle.
   (b) half of one branch of a hyperbola.
   (c) the graph of a function $y = f(x)$.
4. Combinations of functions
   (a) Define combinations of functions for sums, scalar products, vector products and norm.
   (b) Given a set of functions and a combination, evaluate the formula for the combination function. State the domain of the combination function.
5. Vector calculus
   (a) Define the limit of a vector function.

(b) Discuss the geometric interpretation of a limit of a vector function as a limiting direction and a limiting magnitude.

(c) Define continuity for vector function.

(d) Discuss whether the continuity of the norm of a vector function implies the continuity of the vector function. Is the converse true?

(e) Define the derivative and integral of vector functions and work an example of each.

(f) State the differentiation rules and verify at least one of them

6. Tangent vector.

   (a) Motivate the definition of a tangent vector of a curve graphically by analyzing small difference vectors.

   (b) Define the tangent vector of a vector function at a point.

   (c) Define the unit tangent.

   (d) Given a curve in 2-space, parameterize the curve, and then find a tangent vector. Calculate the unit tangent. Is the tangent vector the same for all parameterizations? Is the unit tangent vector the same for all parameterizations?

7. Length of a curve

   (a) Define the length of a curve.

   (b) Derive the arc length formula.

   (c) Calculate the arc length of a piece of a circular helix.

   (d) Define the arc length function.

   (e) Write the arc length function for a circular helix.

   (f) Show that the derivative of the arc length function is the norm of the derivative of the associated vector function.

22 Curvilinear Motion

A. Sketch the curve determined by a vector function in 2-space or 3-space.
B. Parameterize a curve in 2-space or 3-space.
C. Given the position vector function of a moving object, calculate the velocity, speed, and acceleration of the object.
D. Model and analyze curvilinear motion in applications.

Reading: Multivariable Calculus 1.7

Homework: 1.7: 1be,5,7,13,20,D1,D2

Outcome Mapping:

A. D1
B. D2
C. 1
D. 2-22

Lecture:

1. Mechanics
   (a) Define position vector, velocity and acceleration physically.
      i. Position vector - represents the distance and direction from the origin.
      ii. Velocity vector - represents the speed and direction of motion.
      iii. Acceleration vector - represents the instantaneous change in velocity.
   (b) Define position vector, velocity and acceleration mathematically.
2. Uniform circular motion
   (a) Introduce a vector-valued function that represents uniform circular motion.
   (b) Calculate the velocity and acceleration functions for uniform circular motion. Is the speed of the object changing? Why is the acceleration not zero?
   (c) Define angular speed and period. Evaluate for an example.
   (d) Define centripetal and centrifugal acceleration. Evaluate for an example.
   (e) Prove that for uniform circular motion the acceleration vector is perpendicular to the velocity vector.
3. Model a complex physical motion with a vector function and then analyze the motion.

23 Curvature

A. Recall the definitions of unit tangent, unit normal, binormal and osculating plane for a space curve. Illustrate each graphically.
B. Calculate the curvature, the radius of curvature, the center of curvature (*), and the osculating plane for a space curve.
C. Derive formulas for the curvature of a parameterized curve and the curvature of a plane curve given as a function.
D. Determine the tangential and normal components of acceleration for a given path.

Reading: Multivariable Calculus 1.8

Learning Module: Moving Trihedron

Homework: 1.8: 1ace,2bc,3,5bd,8,E1,E2

Outcome Mapping:

A. E1,E2,8
B. 1,5
C. 3,4,7
D. 2

Lecture:

1. Introduce curvature as a measurement of variation of a curve that is not represented in straight line motion and torsion as a measurement of variation of a curve that is not represented in planar motion. Relate each to accelerations due to change in direction of motion.
2. Curvature
   (a) Define the unit tangent. Illustrate with an example.
   (b) Define the curvature as the derivative of the unit tangent vector with respect to arc length.
   (c) Relate the curvature to the radius of curvature, the radius of the circle that best approximates a curve at a point.
   (d) Calculate the curvature of a circular helix.
3. Principal Normal
   (a) Define the principal normal.
   (b) Show that the unit tangent and the principal normal are perpendicular.
   (c) Discuss why the principal normal is defined in terms of the unit tangent and not any arbitrary tangent function.
4. Graphical Representations
   (a) Graphically represent:
      i. the unit tangent
      ii. the unit principal normal
      iii. the unit binormal
   (b) Define the osculating plane. Work an example of finding an osculating plane.
   (c) Describe what is meant by the circle of curvature and the center of curvature. Illustrate with an example.
5. Components of acceleration
   (a) Define the components of acceleration
      i. Normal acceleration - represents the instantaneous rate of change of the direction of the velocity vector. Normal acceleration is in the direction perpendicular to the direction of motion.
      ii. Tangential acceleration - represents the instantaneous rate of change of speed in the direction of the motion.
   (b) Apply the chain rule to decompose acceleration into its components. Discuss the meaning of the coefficients.
   (c) Find the normal and tangential components of acceleration in an example.
6. Derive and apply the curvature formula for a parameterized curve in terms of the velocity and acceleration vectors.

24 Functions of Several Variables

A. Identify the domain and range of a function of several variables.
B. Represent a function of two variables by level curves or a function of three variables by level surfaces.
C. Identify the characteristics of a function from its graph or from a graph of its level curves (or level surfaces).
D. Represent combinations of multivariable functions algebraically.

Reading: Multivariable Calculus 2.1

Homework: 2.1: 1bdfhk,2,5,6bdfh,7aceh,8c,9bdfh

Outcome Mapping:

A. 1
B. 2,4,7,8
C. 3,5,6
D. 9

Lecture:

1. Examples of functions of several variables:
   (a) Write functions that represent the following:
      i. The temperature at points in the xy-plane is proportional to the square of the distance of the point to the origin. Write the temperature as a function of x and y.
      ii. The ideal gas law states that the pressure of a gas if proportional to its temperature and inversely proportional to its volume. Write a function the represents the pressure of a gas in terms of its temperature and volume.

   (b) Evaluate data points and plot the points for the functions in 3-space.
2. Find the domain and range of the following functions:
   (a) The two functions above.
   (b) $f(x, y) = \frac{1}{\sqrt{1-(x^2+y^2)}}$
   (c) $f(x, y, z) = \frac{1}{\sqrt{1-(x^2+y^2+z^2)}}$
   (d) $f(x, y, z) = \cos(\frac{x}{y-z})$
3. Analyze a topographical map - regions of steepness, peaks, valleys, directions of steepest ascent at a point.
4. Introduce the graphing of level curves as a method for understanding the graph of a function in 3-space by a representation in 2-space.
   (a) Graph the level curves of $f(x, y) = x^2 + y^2$ for integer values 0 through 4.
   (b) Discuss the characteristics of the graph of $f(x, y)$ based on the level curves.
   (c) Graph the function $f(x, y)$ in 3-space and compare to predictions.
   (d) Repeat (a)-(c) for $f(x, y) = xy$.
5. Match a sketch of the level curves to the graphs and formulas of the following functions:
   (a) $f(x, y) = 4x^2 - 9y^2$
   (b) $f(x, y) = e^{x^2+y^2}$
   (c) $f(x, y) = \sin x \cos y$
   (d) $f(x, y) = \sin x^2 + y^2$
   (e) $f(x, y) = x^2 e^{-x}$
6. Introduce the graphing of level surfaces as a method for understanding the graph of a function in 4-space by a representation in 3-space.
   (a) graph the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$ for integer values 0 through 4.
   (b) interpret $f(x, y, z)$ as a temperature function and discuss the characteristics of the graph of $f(x, y, z)$ based on the level surfaces.
   (c) repeat (a)-(b) for $f(x, y, z) = \frac{16}{\sqrt{y^2+z^2}}$
7. Use the idea of slicing to illustrate how 4 dimensional objects may be understood.

25 Limits and Continuity

A. Describe a delta neighborhood of a point in 2- or 3-space.
B. Evaluate the limit of a function of several variables for a given approach or show that it does not exists.
C. Determine whether a function is continuous at a given point. Interpret the definition of continuity of a function of several variables graphically.
D. Determine whether a set in 2- or 3-space is open, closed or neither. Determine whether a set is compact.
E. Recall and apply the Extreme Value Theorem.

Reading: Multivariable Calculus 2.2

Homework: 2.2: 1a, 2bdfhj,3aceg,7,8ac,F1,F2,F3,F4

Outcome Mapping:

A. F1
B. 1-7
C. 8,F4
D. F3,10,11,12,13 (3.1:5)
E. F2,9

Lecture:

1. Review definition of the limit of a function of a single variable.
2. Basic topology
   (a) Define the following:
      i. a δ-neighborhood in 2- and 3-space
      ii. an open set
      iii. a closed set
      iv. Can a set be both open and closed? Can a set be neither open nor closed?
   (b) Identify examples of sets as open, closed, neither or both.
3. Limits
   (a) Define a deleted neighborhood of a point.
   (b) Define the limit of a real-valued function of several variables at a point where the function is defined on some deleted neighborhood.
   (c) Present a graphical representation of a function $f(x, y)$ that is
      i. continuous
      ii. not continuous
   (d) Define an approach and a limit along an approach.
   (e) Demonstrate how to evaluate the limit along an approach.
   (f) Show that $f(x, y) = \dfrac{xy + y^3}{x^2 + y^2}$ does not have a limit at $(0, 0)$ by evaluating the limit along straight lines through the origin.
   (g) Show that $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$ has the same limit at $(0, 0)$ along every straight line approach through the origin, but does not have a limit. (Consider the approach $y = x^2$.)
4. Continuity
   (a) Define continuity for a function of several variables.
   (b) Present classes of functions that are continuous.
      i. polynomials, trig, exponential and logarithm
      ii. rational functions (with restricted domain if the denominator is ever zero)
      iii. combinations of continuous functions
   (c) Identify a function as continuous at a given point by identifying the function as continuous at the point.
5. Extreme Value Theorem

(a) Define boundedness. Give and example of a bounded set without boundary and an unbounded set with boundary. Contrast the idea of boundedness with boundary.

(b) Define compact. Have students identify examples of sets that are compact and are not compact.

(c) State the extreme value theorem. Illustrate the meaning of the extreme value theorem graphically. Why is compactness necessary?

(d) Explain that we will be using the extreme value theorem to find maximum and minimum values of a function over a compact domain.

26 Partial Derivatives

A. Interpret the definition of a partial derivative of a function of two variables graphically.
B. Evaluate the partial derivatives of a function of several variables.
C. Describe the relationship between the existence of partial derivatives and the existence of a derivative for a function of several variables.
D. Evaluate the higher order partial derivatives of a function of several variables.
E. State the conditions under which mixed partial derivatives are equal.
F. Verify equations involving partial derivatives.
G. Evaluate the gradient of a function.
H. Prove identities involving the gradient.

Reading: Multivariable Calculus 2.3

Homework: 2.3: 1,4,5behk,6ac,7,9behk,10bc,15a,G1,G2,G3

Outcome Mapping:

A. G1,4,11,18
B. 2,3,5,6,19 (2.4:2)
C. G2
D. 5,7,8
E. G3
F. 12-17
G. 9
H. 10

Lecture:

1. Review definition of derivative of a function of a single variable as a limit.
2. Partial derivatives of functions of two variables.
   (a) Define partial derivatives of a function of two variables. Compare to the derivative of a function of a single variable.
   (b) Emphasize that it follows from the definition that the partial derivatives of a function of two variables can be evaluated by treating the other variable as a constant. Work an example.
   (c) Graphically illustrate the meaning of a partial derivative of a function of two variables.
3. Partial derivatives of functions of three variables.
   (a) Define partial derivatives of functions of three variables.
   (b) Demonstrate how to compute partial derivatives of functions of three variables.
   (c) Consider a function of three variables as a temperature function. Interpret the partial derivatives of a function of three variables as the rate of change in temperature while moving parallel to a coordinate axis.
4. Introduce the ∂ notation for partial derivatives. Contrast with the subscript notation. Emphasize that the ordering of the derivative symbols in the notations are different.
5. Higher order derivatives
   (a) Define derivatives of higher order including mixed partials. Make clear that the order of partial derivatives is indicated in the notation.
   (b) Give conditions under which mixed partials are equal and check a few examples.
6. Relationships between continuity and partial differentiability
   (a) Review continuity for a function of a single variable and how differentiability at a point implies continuity at the point.
   (b) Show that the existence of partial derivatives at a point does not imply continuity at the point.
7. Gradient
   (a) Define gradient.
   (b) Work examples of evaluating a gradient.
   (c) Introduce gradient formulas.
   (d) Verify the product formula for the gradient.

27 Differentiability and the Chain Rule

A. Define differentiability for a function of several variables.
B. Evaluate partial derivatives from the definition. Describe the relationship between the derivative of a multivariable function and its partial derivatives.
C. Apply the chain rule to evaluate derivatives.
D. Solve related rates problems using the chain rule.

Reading: Multivariable Calculus 2.4

Homework: 2.4: H1,1,3b,5,6ac,8

Outcome Mapping:

A. H1
B. H1,1
C. 3,5,6
D. 4,7,8,9

Lecture:

1. Definition of a derivative
   (a) Review the definition of the derivative of a function of a single variables.
   (b) Rewrite it into a form that can be generalized for functions of several variables.
   (c) Define the derivative for a function of several variables.
2. Evaluation of derivatives
   (a) Find the derivative of a function of several variables from the definition. Observe that the derivative is the same as the gradient.
   (b) Prove that the gradient of a function is the derivative provided that the derivative exists, i.e., the partial derivatives are continuous.
3. The chain rule
   (a) Derive the chain rule for a function of two variables defined in terms of a function of a single variable.
   (b) Use tree diagrams to write general chain rules.
   (c) Work examples of applying chain rules to word problems.

28 Directional Derivative

A. Give a graphical interpretation of the gradient.
B. Evaluate the directional derivative of a function.
C. Give a graphical interpretation of directional derivative.
D. Prove that a differential function f increases most rapidly in the direction of the gradient (the rate of change is then $\|f(\vec{x})\|$) and it decreases most rapidly in the opposite direction (the rate of change is then $-\|f(\vec{x})\|$).
E. Find the path of a heat seeking or a heat repelling particle.

Reading: Multivariable Calculus 2.5

Homework: 2.5: 1,3,4bf,5,8,10c,H2,H3

Outcome Mapping:

A. 1,11,H2 (2.6:2)
B. 4,6,10
C. 2,3,H3 (2.6:2)
D. H4
E. 5,7,8,9

Lecture:

1. Definition of directional derivatives
   (a) Review the definitions of the partial derivatives of a function three variables.
   (b) Give definition of partial derivatives in terms the the unit vectors $\vec{i}$ and $\vec{j}$.
   (c) Define what is meant by a directional derivative.
   (d) Graphically illustrate the meaning of the directional derivative.
   (e) Write the partial derivatives using the directional derivative notation.
2. Evaluation of directional derivatives
   (a) Prove that the directional derivative is the dot product of the gradient with the unit direction vector.
   (b) Work examples of evaluating directional derivatives.
3. Applications of the directional derivative
   (a) Prove the relationship between the direction of maximum (or minimum) increase of a function and the magnitude of maximum (or minimum) increase of the function with the gradient of the function.
   (b) Relate the path of a heat seeking or heat repelling particle to the gradient of a temperature function. Work examples of finding path of heat seeking and heat repelling particles.

29 Normal Vectors and Tangent Planes

A. Interpret the gradient of a function as a normal to a level curve or a level surface.
B. Find the normal line and tangent plane to a smooth surface at a given point.
C. Find the angles between curves and surfaces.

Reading: Multivariable Calculus 2.6

Homework: 2.6: 1cg,3d,6,9,11,14,15b,16,17a,18b

Outcome Mapping:

A. 1,3,11,12,13
B. 4-11
C. 14,15,16,17

Lecture:

1. Prove that the gradient of a function of two variables at a given point is orthogonal to the level through the given point. Discuss the analogous result for functions of three variables and level surfaces.
2. Sketch several level curves of $f(x, y) = x^2 + y^2$. Evaluate the gradient and several points to verify the the above result is correct.
3. Use gradients to determine the following for a surface:

   (a) normal lines

   (b) tangent planes

   (c) angle between two surfaces

   (d) angle between a curve and a surface

30 Extrema of Functions of Several Variables

A. Identify local extreme values graphically.
B. Determine the local extreme values and saddle points of a function of two variables. When possible, apply the second partial derivatives test.
C. Identify the extreme values of a function defined on a closed and bounded region.
D. Solve word problems involving maximum and minimum values.

Reading: Multivariable Calculus 2.7

Homework: 2.7: 1,3bgmo,4ad,7,12,14

Outcome Mapping:

A. 1,2
B. 3
C. 4,5
D. 6-25

Lecture:

1. Definitions
   (a) Define local maximum, local minimum and saddle points. Present graphic examples.
   (b) Graphically observe the value of the gradient at local extreme values.
   (c) Define critical points and stationary points.
2. Analysis
   (a) Determine the extreme values of a function of two variables. Classify the extreme values using the criteria given in the definition.
   (b) State the second partial derivatives test. Discuss the meaning of the second partial derivatives and the discriminant.
   (c) Apply the second partial derivatives test to identify the type of local extreme values.
   (d) Set up and solve application problems involving local extreme values.
3. Optimization over compact sets
   (a) Define extreme values for a function defined on a compact domain.
   (b) Recall the extreme value theorem.
   (c) Emphasize that extreme values are found at critical values of the surface or along the boundary.
   (d) Solve extreme value problems in the case of a function defined over (a) a disk and (b) a triangle.

31 Constrained Extrema

A. Graphically interpret the method of Lagrange.
B. Determine the extreme values of a function subject to a side constraints by applying the method of Lagrange.
C. Apply the method of Lagrange to solve word problems.

Reading: Multivariable Calculus 2.9

Homework: 2.9: 1,3aci,4,6,13

Outcome Mapping:

A. 1,2
B. 3
C. 4-20

Lecture:

1. Derivations
   (a) Illustrate graphically why the gradient of the constraint function is in the same direction as the gradient of the function at a local extreme value.
   (b) Prove that at a local extreme value the gradient of the constraint function is in the same direction as the gradient of the function.
   (c) Define the Lagrange function and the system of partial derivatives used to determine local extreme values.
2. Apply the method of Lagrange to solve a couple of word problems.
3. Multiple constraints
   (a) Define the Lagrange for a problem with multiple constraints.
   (b) Illustrate the method of Lagrange in this case with an example.

32 Double Integrals

A. Compare the definition of the double integral to the method of repeated integration geometrically.
B. Evaluate double integrals over a rectangle by repeated integration.
C. Apply a double integral to calculate the volume or mass of a solid.

Reading: Multivariable Calculus 3.1

Homework: 3.1: 1bf,2abcdg,3c,4b,6,J1

Outcome Mapping:

A. J1
B. 1,2,6
C. 3,4

Lecture:

1. Summation notation
   (a) Review summation notation for a single sum.
   (b) Introduce double and triple sums using sigma notation.
   (c) Expand out a double and triple sum given in sigma notation.
2. Double integrals
   (a) Define the double integral over a rectangle.
   (b) Interpret the double integral of a positive function as a volume and any double integral as the sum of signed volumes.
   (c) Illustrate the slice method for computing volume graphically. Relate to the method of repeated integration.
   (d) Evaluate several double integrals by repeated integration.
3. Applications
   (a) Derive the integral formula for the mass of a solid with varying density.
   (b) Find the mass of a solid with varying density by using integration.

33 Double Integrals over General Regions

A. Evaluate double integrals over general regions.
B. Evaluate double integrals by interpreting them as known volumes.
C. Rewrite a double integral changing the order of integration.
D. Apply double integrals to calculate areas of planar regions and volumes of solids.
E. Evaluate the physical characteristics of a plate such as mass, centroid, center of mass and moment of inertia.

Reading: Multivariable Calculus 3.2

Homework: 3.2: 1bd,2bc,3ab,4be,5b,6b,7a,8

Outcome Mapping:

A. 1
B. 2
C. 3
D. 4,8
E. 5,6,7,8

Lecture:

1. Formulas
   (a) Review the method of repeated integration over a rectangle.
   (b) Define the double integral over a general region in terms of a Riemann sum.
   (c) Define what is meant by a Type x and a Type y region.
   (d) Illustrate the Slice Method for evaluating integrals over Type x and Type y regions.
2. Setting up an integral
   (a) Show how to set up limits for repeated integrals.
   (b) Given an integral, sketch the region of integration
   (c) Demonstrate how to change the order of integration.
3. Applications
   (a) Derive the integral formulas for centroid, center of mass and moment of inertia.
   (b) Work examples on finding centroids, centers of mass and moments of inertia.

34 Double Integrals in Polar Coordinates

A. Represent a region in both Cartesian and polar coordinates.
B. Evaluate double integrals in polar coordinates.
C. Convert a double integral in Cartesian coordinates to a double integral in polar coordinates and then evaluate.
D. Evaluate areas and volumes using polar coordinates
E. Evaluate the physical characteristics of a plate such as centroid, mass, and center of mass using polar coordinates.
F. Make conversions of algebraic expressions between Cartesian coordinates and cylindrical coordinates.

Reading: Multivariable Calculus 3.3

Homework: 3.3: 2,3,5a(iii)b(i,iii,iv)c(ii),7a,8ac,9bd,10ad,11abce,12abf

Outcome Mapping:

A. 1,2,3,4
B. 5a
C. 8
D. 5b,6,7
E. 5c
F. 9,10,11,12

Lecture:

1. Review polar coordinates.
2. Write formulas in Cartesion coordinates in terms of polar coordinates and vice versa.
3. The double integral in polar coordinates
   (a) Define Riemann Sums. Show that they are always sandwiched between lower and upper sums.
   (b) Show that $r\Delta r\Delta\theta$ represents the area of a wedge where r is the average of the outer radius and the inner radius. Use this to explain the substitution $dxdy = rdrd\theta$. Explain r as the *scaling factor* as you change variables.
   (c) Write the double integration formula in terms of polar coordinates. (Stress that an algebraic conversion of limits does not work. The geometry of the region over which the function is integrated must be understood.)
   (d) Work examples of evaluating double integrals by using polar coordinates.
4. Applications
   (a) Review calculation of area and centroid using thin rectangular strips.
   (b) Compute area and mass using double integrals. Work a few examples.
   (c) Compute the center of mass of a plate of variable density by computing the moment about the x-axis and the y-axis. Work a few examples.

35 Triple Integrals

A. Find the volume of a solid using triple integration in Cartesian coordinates.
B. Evaluate the physical characteristics of a solid such as mass, centroid and center of mass using Cartesian coordinates.

Reading: Multivariable Calculus 3.4

Homework: 3.4: 1cei,3,7,9

Outcome Mapping:

A. 1,3,4,7,8,9
B. 2,3,4,5,6,7,8,9

1. Define the triple integral over a solid in 3-space by upper sums and lower sums. Evaluate a simple triple integral from the definition.
2. Interpret the definition of the triple integral a the mass of a solid with variable density.
3. Use mass of a solid with variable density and slices to motivate the evaluation of a triple integral by repeated integration.
4. Work examples of evaluating triple integrals by repeated integration.
5. Evaluate a repeated integral by interpreting it as a triple integral and changing the order of integration.

36 Triple Integrals in Cylindrical Coordinates

A. Describe regions in both Cartesian coordinates and cylindrical coordinates.
B. Evaluate triple integrals using cylindrical coordinates.
C. Find volumes by applying triple integration in cylindrical coordinates.
D. Evaluate the physical characteristics of a solid such as mass, centroid and center of mass using cylindrical coordinates.

Reading: Multivariable Calculus 3.5

Homework: 3.5: 1,3,5b,6b,7a,8,11bd,12

Outcome Mapping:

A. 1,2,34
B. 5,11
C. 6,9,10,12
D. 7,8,12

Lecture:

1. Review polar coordinates including switching back and forth from polar to Cartesian coordinates.
2. Introduce cylindrical coordinates and how to change between Cartesian and cylindrical coordinates. Convert coordinate of points and equations.
3. Determine the limits of integration over a solid in cylindrical coordinates.
4. Compute the volume of a 3-dimensional wedge and use it to explain the substitution $dxdydz = rdrd\theta dz$.
5. Evaluate integrals using cylindrical coordinates. Compute a volume and a center of mass.
6. Convert integrals in Cartesian coordinates to integrals in cylindrical coordinates.

37 Triple Integrals in Spherical Coordinates

A. Describe regions in both Cartesian coordinates and spherical coordinates.
B. Evaluate triple integrals using spherical coordinates.
C. Find volumes by applying triple integration in spherical coordinates.
D. Evaluate the physical characteristics of a solid such as mass, centroid and center of mass using spherical coordinates.
E. Convert a triple integral in Cartesian coordinates to cylindrical or spherical coordinates and then evaluate.
F. Make conversions of algebraic expressions between Cartesian coordinates and spherical coordinates.

Reading: Multivariable Calculus 3.6

Homework: 3.6: 1,3,5a,6ad,7a,10ab,11,12ad,13ad,14ad,15bce,16bde,19

Outcome Mapping:

A. 1,2,3,4
B. 5,10
C. 6,9,11
D. 7,8,11
E. 10
F. 12-19

Lecture:

1. Introduce spherical coordinates and how change between Cartesian and spherical coordinates. Convert coordinate of points and equations.
2. Compute the volume of a spherical wedge and use it to explain the substitution $dxdydz = \rho^2 \sin\phi d\rho d\theta d\phi$.
3. Determine the limits of integration over a solid in spherical coordinates.
4. Evaluate integrals using spherical coordinates. Compute a volume and a centroid.
5. Convert integrals in Cartesian and cylindrical coordinates into integrals in spherical coordinates.

38 The Jacobian

A. Find the Jacobian of a transformation.
B. Change variables in a multiple integration to obtain a more simple integral and then evaluate.

Reading: Multivariable Calculus 3.7

Homework: 3.7: 1aek,3a,4b,5

Outcome Mapping:

A. 1,2,7
B. 3,4,5,6,8,9

Lecture:

1. Introduce the $r\theta$-plane and the polar map to the xy-plane. Explain that the scaling factor for changing a $\Delta r\Delta\theta$ to a $\Delta x\Delta y$ region is r.
2. Introduce the uv-plane and a map to the xy-plane. Explain that if the map is a linear map, then the scaling factor is the absolute value of the determinant.
3. Explain that since maps can be approximated by linear maps, then the scaling factor is the absolute value of the Jacobian.
4. Explain the change of variables formula for area.
5. Explain the change of variables formula for integrals.
6. Explain that polar, cylindrical, and spherical coordinates are special cases of the change of variables formula.

39 Recovering a Function from Its Gradient

A. Analyze the characteristics of a vector field. Sketch a vector field.
B. Determine whether or not a vector field is a gradient.
C. Determine whether or not a differential form is exact.
D. Recover a function from its gradient or differential form, if possible.

Reading: Multivariable Calculus 4.1

Homework: 4.1: 2,4achkl,5ag,7,8a

Outcome Mapping:

A. 1,2,3,7,8
B. 5,8,12
C. 4,6,7
D. 4,5,8,12-16

Lecture:

1. Discuss the differences between a vector function a scalar function and a vector field.
2. Illustrate how to graph a vector field. Explain why other representations are not as effective.
3. Review how to calculate the gradient of a function
4. Illustrate how to recover a potential function from its gradient.
5. Illustrate how the above process may fail.
6. Introduce the necessary and sufficient condition for the existence of potential function.
7. Define exactness for differentials.
8. Give examples of differentials that are exact and not exact. Recover the potential function if possible.

40 Line Integrals

A. Evaluate the work done by a varying force over a curved path.
B. Evaluate line integrals in general including line integrals with respect to arc length.
C. Evaluate the physical characteristics of a wire such as centroid, mass, and center of mass using line integrals.

Reading: Multivariable Calculus 4.2

Homework: 4.2: 1ach,2ac,3b,4ab,6,8

Outcome Mapping:

A. 1,8
B. 2,3,4,7
C. 5,6

Lecture:

1. Review/Introduction
   (a) Line integral is an extension of idea of definite integral over an interval to an integral over a curve.
2. Line Integral
   (a) Definition: A line integral of a vector function $\mathbf{F}$ over a curve C given by a vector function $\mathbf{r}(t), a \le t \le b$ is
   $$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\ dt.$$
   (b) Physical interpretation as work.
   (c) Example.
   (d) Definition: A line integral of a scalar function f over a curve C parameterized by arc length is
   $$\int_C f(\mathbf{r})ds = \int_a^b f(\mathbf{r}(t))\sqrt{(x'(t))^2 + (y'(t))^2}\ \ dt.$$
   (e) Physical interpretation as area of one side of "fence."
   (f) Example.
3. Questions
   (a) Does the value of the line integral depend upon the parameterization of C? No.
   (b) Does the value of the line integral depend upon the path from the initial point to the end point? Yes.

41 Path Independent Line Integrals

A. Recall and apply the Fundamental Theorem for Line Integrals.
B. Determine whether or not a force field is conservative, and if so, find its potential.
C. Evaluate the circulation of a force field or the work done by a force field on a object moving along a given path.

Reading: Multivariable Calculus 4.3

Homework: 4.3: K1,1bdir,2be,3cg,4ab,7

Outcome Mapping:

A. K1,1
B. 3,5,6,7
C. 2,4

1. Review/Introduction
   (a) Recall the Fundamental Theorem of Calculus.
   (b) Extend this idea to line integrals with the gradient representing the idea of derivative.
2. Fundamental Theorem for Line Integrals.
   (a) Theorem 1: $\int_C \text{grad } f \cdot d\mathbf{x} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$.
   (b) Proof of Theorem 1.
   (c) Example.
   (d) The idea of independence of path.
3. Applying the Fundamental Theorem.
   (a) Definition: A differential form $F_1\ dx + F_2\ dy + F_3\ dz$ is exact $\iff$ there exists a differentiable function f such that $\mathbf{F} = (F_1, F_2, F_3)$=grad f, or
   $$F_1 = \frac{\partial f}{\partial x}, \qquad F_2 = \frac{\partial f}{\partial y}, \qquad F_3 = \frac{\partial f}{\partial z}.$$
   (b) Equivalent conditions:
      i. Fundamental Theorem applies;
      ii. Line integral independent of path;
      iii. Differential form is exact;
      iv. curl $\mathbf{F} = \mathbf{0}$ or
      $$\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \qquad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}, \qquad \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}.$$

   In such a case, the vector field $\mathbf{F}$ is called conservative.

42 Green's Theorem

A. Recall and verify Green's Theorem.
B. Apply Green's Theorem to evaluate line integrals.
C. Apply Green's Theorem to find the area of a region.

Reading: Multivariable Calculus 4.4

Homework: 4.4: L1,L2,1adf,2c,3,5

Outcome Mapping:

A. L1,L2,3,5,6
B. 1,7
C. 2,4

Lecture:

1. Review/Introduction
   (a) Green's Theorem provides a relationship between line integrals and double integrals; it can be thought of as the analogue of the Fundamental Theorem of Calculus for double integrals.
   (b) This allows us to evaluate certain integrals more easily.
2. Green's Theorem
   (a) Green's Theorem: $\iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \quad dx\ dy = \int_C (F_1\ dx + F_2\ dy).$
   (b) Example.
   (c) Proof (for special case).
3. Application of Green's Theorem
   (a) Area of a plane region as a line integral: $A = \frac{1}{2}\int_C (x\ dy - y\ dx).$
   (b) Double integral of the Laplacian equals the line integral of its normal derivative:

$$\iint_R \nabla^2 w\ dx\ dy = \int_C \frac{\partial w}{\partial n}\ ds.$$

43 Surface Integral

A. Determine the area of a given surface using integration.
B. Evaluate the physical characteristics of a surface such as centroid, mass, and center of mass using surface integrals.
C. Find the flux of a vector field through a surface.

Reading: Multivariable Calculus 4.5

Homework: 4.5: 1afi,2d,3a(ii),c(ii),d(iii)

Outcome Mapping:

A. 1,5
B. 1,2,4,6
C. 3

Lecture:

1. View a surface as the image of a region in the xy-plane.
2. Relate an element of area in the xy-plane to the corresponding area in the surface.
3. Derive the formula for an element of surface area in terms of an element of area of the domain.
4. Rewrite a surface integral as an integral over a region in the xy-plane
5. Demonstrate how to find the mass and center of mass of a surface.
6. Define Flux through a flat surface.
7. Derive the flux integral for a vector field through a surface.
8. Evaluate flux integrals.

44 Parametric Surfaces

A. Write a parameterization for a given surface.
B. Identify a surface from its parameterization.
C. Describe a surface from its nets. Sketch a parametric surface.

Reading: Multivariable Calculus 4.6

Homework: 4.6: 1,3,4,5,6cd,7b,8

Outcome Mapping:

A. 3,5,8,9 (4.7: 7,9)
B. 1,2,4 (4.7:10)
C. 6,7

1. Review/Introduction
   (a) Recall some surfaces we have studied (e.g., sphere, cylinder, cone, paraboloid, etc.).
   (b) We have seen that we can describe a curve by a vector function $\mathbf{r}(t)$ of one variable. Similarly, we can describe a surface by a vector function $\mathbf{r}(u, v)$ of two variables.
   (c) Such a parameterization will allow us to handle surface integrals in which we integrate a function over a surface.
2. Representation of Surfaces
   (a) Parametric representation: $\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$.
   (b) Example.
3. Identify surfaces from their parameterization.
4. Parameterize a given surface.
5. Describe a set of nets for the surface.
6. Graph the nets for a surface.

45 Integrals Over Parametric Surfaces

A. Graphically describe a surface in terms of its parameterization.
B. Determine a (unit) normal vector to a surface from a parameterization of the surface.
C. Determine the plane tangent to a surface at a given point.
D. Evaluate the physical characteristics of parameterized surfaces such as centroid, mass, and center of mass.
E. Find the flux of a flow through a parametric surface.

Reading: Multivariable Calculus 4.7

Homework: 4.7: 2,4

Outcome Mapping:

A. 1-6
B. 1-6
C. 1-6
D. 1-6,7,8,9
E. 1-6,7,9

Lecture:

1. Tangent Plane and Surface Normal.
   (a) Definition: The tangent plane of a surface S at a point P is the plane containing all the tangent vectors of curves on S through P.
   (b) Definition: A normal vector of S at P is a vector orthogonal to the tangent plane of S at P.
   (c) Formula for normal vector: $\mathbf{N} = \mathbf{r}_u \times \mathbf{r}_v \neq \mathbf{0}$.
   (d) Unit normal vector $\mathbf{n}$ is $\mathbf{n} = \dfrac{\mathbf{N}}{|\mathbf{N}|} = \dfrac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$.
   (e) Example.
2. Review/Introduction.
   (a) We can use a parameterization of a surface to define the concept of a surface integral in which we integrate either a vector function or a scalar function over a surface.
3. Surface Integrals of Vector Functions.
   (a) Derive the flux integral formula for parameterized surfaces

$$\iint_S \mathbf{F} \cdot \mathbf{n}\ \ dA = \iint_R \mathbf{F}[\mathbf{r}(u,v)] \cdot \mathbf{N}(u,v)\ du\ dv.$$

   (b) Example.
4. Surface Integrals of Scalar Functions.
   (a) Definition: Surface integral of a scalar function G over a surface S is

$$\iint_S G(\mathbf{r})dA = \iint_R G(\mathbf{r}(u,v))\ |\mathbf{N}(u,v)|\ du\ dv.$$

   (b) Example.
   (c) Application: If $G = 1$, then this surface integral gives the surface area of S:

$$A(S) = \iint_S dA = \iint_R |\mathbf{r}_u \times \mathbf{r}_v|\ du\ dv.$$

46 Flux Density and Divergence

A. Explain what is meant by the flux density and divergence of a vector field.
B. Evaluate the divergence of a vector field.
C. Evaluate the Laplacian of a function.
D. Derive formulas involving divergence, gradient and Laplacian.

Reading: Multivariable Calculus 5.1

Homework: 5.1: M1,1abcdej,2,3afg,4abc

Outcome Mapping:

A. M1
B. 1
C. 2,3
D. 4,5

1. Review vector field.
2. Divergence
   (a) Definition: div $\mathbf{v} = \dfrac{\partial v_1}{\partial x} + \dfrac{\partial v_2}{\partial y} + \dfrac{\partial v_3}{\partial z}$.
   (b) Alternate formula: div $\mathbf{v} = \nabla \cdot \mathbf{v}$.
   (c) Example.
   (d) Physical interpretation: divergence of $\mathbf{v}$ at a point p corresponds to the net flow of fluid of a small box centered at p; incompressibility means "div $\mathbf{v} = 0$".
3. Laplacian
   (a) Review Laplace's equation: $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0$.
   (b) Laplacian: div (grad f)$= \nabla^2 f$.
   (c) Example.
4. Prove identities involving divergence.

47 The Divergence Theorem

A. Recall and verify the Divergence Theorem.
B. Apply the Divergence Theorem to evaluate the flux through a surface.
*C. Derive integration formulas using the Divergence Theorem.

Reading: Multivariable Calculus 5.2

Homework: 5.2: N1,N2,1ceh,2,4,5,8

Outcome Mapping:

A. N1,N2,4,8
B. 1,2,3
*C. 5,6,7

Lecture:

1. Review/Introduction
   (a) Review the idea and meaning of divergence of a vector field.
   (b) The Divergence Theorem provides a relationship between surface integrals and triple integrals.
   (c) This allows us to evaluate certain integrals more easily.
2. Divergence Theorem
   (a) Divergence Theorem: $\iiint_T \text{div } \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot \mathbf{n} \; dA$.
   (b) Example.
   (c) Proof (for special case).
3. Application of Divergence Theorem.
   (a) Volume of a surface as a surface integral: $V = \frac{1}{3} \iint_S (x \, dy \, dz + y \, dz \, dx + z \, dx \, dy)$.

48 Circulation Density and Curl

A. Explain what is meant by the circulation density and curl of a vector field.
B. Evaluate the curl of a vector field
C. Derive and apply formulas involving divergence, gradient and curl.

Reading: Multivariable Calculus 5.3

Homework: 5.3: O1,1bej,2abcd,3,4,6

Outcome Mapping:

A. O1,3,7
B. 1,4,5,6
C. 2

Lecture:

1. Review Divergence
2. Introduce circulation density
3. Curl
   (a) Definition: curl $\mathbf{v} = \nabla \times \mathbf{v} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{bmatrix}$.
   (b) Physical interpretation: curl of $\mathbf{v}$ at a point p gives a vector parallel to the axis of rotation of the flow lines near p.
   (c) Connection with irrotational motion and conservative vector fields.
4. Formulas involving grad, div, and curl
   (a) curl(grad f)= $\mathbf{0}$.
   (b) div(curl v)= 0.
5. Compare the curl vector to the gradient vector.

49 Stoke's Theorem

A. Recall and verify Stoke's theorem.
B. Apply Stoke's theorem to calculate the circulation (or work) of a vector field around a simple closed curve.
C. Recall and apply the divergence and curl tests.

Reading: Multivariable Calculus 5.4

Homework: 5.4: P1,P2,1abcd,2a,3a,4d,10,12dh

Outcome Mapping:

A. P1,1,6,11,12
B. 2,3,7,8,9,13
C. P2,4,5,10

Lecture:

1. Review/Introduction
   (a) Review the idea and meaning of curl of a vector field.
   (b) Stoke's Theorem provides a relationship between line integrals and surface integrals; it can be thought of as an generalization of Green's Theorem.
   (c) This allows us to evaluate certain integrals more easily.
2. Stoke's Theorem
   (a) Stoke's Theorem: $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA = \int_C \mathbf{F} \cdot \mathbf{r}'(s) \, ds$.
   (b) Example.
   (c) Proof (for special case).
3. Summary.
   (a) Summarize relationships among line, surface, double and triple integrals provided by Green's, Divergence, and Stoke's Theorems.

50 Iterative Methods for Solving Linear Systems

Outcomes:

A. Apply Jacobi iteration to approximate a solution to a linear system of equations.
B. Apply Gauss-Seidel iteration to approximate a solution to a linear system of equations.
C. Analyze convergence and divergence in the application of the Jacobi and Gauss-Seidel methods. Use diagonal dominance to determine convergence.
D. Use iterative methods to solve application problems.

Reading: Linear Algebra 2.5

Homework: 2.5: 1,4,7,10,22,28

Outcome Mapping:

A. 1-6
B. 7-12
C. 13-14,15-17,18-21
D. 22-28

Lecture:

1. Explain that the general strategy for iterative methods is to start from an approximation of the true solution and obtain better and better approximations from a computational cycle.
2. Motivation for using indirect or iterative methods:
   (a) saves operations if convergence is fast
   (b) saves computer storage space for sparse systems, i.e. systems involving a large number of zero coefficients.
3. Work an example of Guass-Seidel iteration.
4. Derive the iteration formula for Guass-Seidel iteration.
5. Discuss the algorithm for Guass-Seidel iteration.
6. Explain the strategy of Jacobi iteration.
7. Work an example of Jacobi iteration.

51 Numerical Methods for Solving the Eigenvalue Problem

Outcomes:

A. Use the power method to approximate the dominant eigenvalues and eigenvectors.
B. Identify conditions under which the power method applies.
C. Use the Rayleigh quotient method to approximate dominant eigenvalues.
D. Use the shifted power method, the inverse power method and the shifted inverse power method to find the non-dominant eigenvalues and eigenvectors.
*E. Construct the Gershgorin disks for a matrix. Relate the Gershgorin disks to the eigenvalues of a matrix.

Reading: Linear Algebra 4.5

Homework: 4.5: 3,7,10,12,18,30,34,38

Outcome Mapping:

A. 1-4,5-8,9-14,15-16
B. 21-24,25-28
C. 17-20
D. 29-32,33-36,37-40,41-45
*E. 47-50,51