Applied Math Syllabus for PhD Qualify Exam

Math 634 and Math 635

1. Existence and Uniqueness of Solutions to Ordinary Differential Equations:
   - Existence of solutions;
   - The contraction mapping principle.
   - Uniqueness of solutions;
   - Gronwall inequality.
   - Continuation of solutions;
   - Dependence of solutions on initial conditions and Parameters;
2. Linear Differential Equations and Linearization:
   - Linear systems with constant coefficients;
   - Jordan normal form of matrices, fundamental solutions.
   - Nonautonomous linear systems;
     - Fundamental metric solutions, the variation of constants formula.
     - Floquet theory for periodic systems
3. Stability of Equilibrium Points
   - Stability and asymptotic stability
   - Lyapunov functions
   - Gradient Systems
   - Phase-plane analysis
4. Poincare-Bendixon Theory
   - Invariant sets, omega-limit sets, alpha-limit sets
   - Limit cycles
   - Van der Pol's equation
5. Stability of Nonautomous Equations
   - Stability, asymptotic stability, uniform stability, uniformly asymptotic stability
   - Stability of linear systems
   - Existence of Periodic orbits
   - Fredholm Alternative
   - Stability of Periodic orbits
   - Orbital stability
   - Poincare section and Poincare maps
6. Stable and Unstable Manifolds
7. Discrete Dynamical Systems
   - Stability of Critical Points
   - Stable and unstable manifolds
   - Structural stability, Hartman-Grobman theorem
   - Smooth conjugacy
   - Bifurcations
   - Dynamical bifurcations, bifurcations from simple Eigenvalues, global bifurcations, Crandall-Robinowitz Theorem, Periodic doubling bifurcations, Feigenbaum universal numbers
   - Symbolic dynamics and Smale Horseshoe maps, transveral homolinic orbits
8. Center Manifolds
   - Hopf Bifurcations
9. Normal form theory
10. Averaging and local bifurcations
11. Homoclinic Bifurcations, Melnikov functions
    - Exponential dichotomy, shadowing lemma
    - Uniform Contraction Principle. Gronwall's Inequality. Variation of Constants Formuala
12. Geometric Theory of Ordinary Differential Equations
    - Stability of Equilibrium, Lyapunov Fuctions, Gradient Systems, Invariant Set, Omega-limit set, Alpha-limit set, Poincare-Bendixon Theorem, Floquet Theory, Existence and Stability of Periodic Solutions, Stable and Unstable Manifold Theorem, Center Manifold Theorem, Hartman-Grobman Theorem,
13. Dynamical Systems Theory.