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Section:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Instructor:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Math 112 (Calculus I)
Final Exam   Form A
December 15, 2009

Instructions:

- Work on scratch paper will not be graded.

- In the multiple choice and short answer sections, **only** the answer will be graded. There will be no partial credit.

- For questions 10 to 20, partial credit will be given if you show **all** your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.

- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.

- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned	#	Possible	Earned
MC	24		15	10	
9	11		16	6	
10	8		17	6	
11	6		18	6	
12	4		19	6	
13	3		20	6	
14	4				
Sub	60		Sub	40	
			Total	100	

Multiple Choice. Fill in the answer to each problem on your computer-scored answer sheet. Make sure your name, section and instructor are on that sheet.

1. Approximate $\displaystyle\int_1^5 x^4\,dx$ using a Left Hand sum with 2 subintervals (n=2).

   (a) 82 (b) 164 (c) 81 (d) 162 (e) 624 (f) 625

   (g) None of these

2. Find the area under the function $f(x) = \sqrt[3]{x}$ from $x = 1$ to $x = 8$.

   (a) $\dfrac{45}{4}$ (b) $\dfrac{1}{4}$ (c) 12 (d) 15 (e) $\dfrac{1}{12}$ (f) None of these

3. Given the limit statement $\displaystyle\lim_{x\to 1}(2x - 3) = -1$ pick the largest δ that works with the definition of the limit if $\epsilon = 0.06$.

   (a) 0.001 (b) 0.005 (c) 0.01 (d) 0.02 (e) 0.03 (f) No such δ exists

4. Which of the following is an inflection point of $f(x) = \dfrac{x}{x^2 + 1}$?

   (a) 1 (b) -1 (c) 2 (d) -2 (e) $\sqrt{2}$ (f) $-\sqrt{2}$ (g) -3 (h) $\sqrt{3}$

5. Given $x\ln y - y\ln x = e^2 - 2e$, find $\dfrac{dy}{dx}$ at the point (e^2, e).

   (a) 0 (b) e (c) e^2 (d) $\dfrac{1-e}{e^2}$ (e) $\dfrac{1-e}{e^2 - 2e}$ (f) $e^2 - 2e$ (g) $\dfrac{e-1}{e^2}$

6. Which of the following are x-values for which $f(x) = \sin(x) - x$ has a local maximum?

   (a) -2π (b) $-\pi$ (c) 0 (d) π (e) 2π (f) More than one of these

   (g) None of these

7. Which of the following functions has a discontinuous first derivative?

   (a) $\sinh(x)$ (b) $x^{1/3}$ (c) $\tan^{-1}(x)$ (d) $\dfrac{x}{1 + x^2}$ (e) $\ln(x^2 + 1)$

   (f) All of the first derivatives of these functions are continuous

8. $\dfrac{d}{dx}\displaystyle\int_1^{2x} \sqrt{1 + t^3}\,dt =$

   a) $\sqrt{1 + (2x)^3} - \sqrt{2}$ b) $2\sqrt{1 + (2x)^3} - \sqrt{2}$ c) $\sqrt{1 + x^3} - \sqrt{2}]$

   d) $2\sqrt{1 + x^3} - \sqrt{2}$ e) $\sqrt{1 + (2x)^3}$ f) $2\sqrt{1 + (2x)^3}$

   g) $\sqrt{1 + x^3}$ h) $2\sqrt{1 + x^3}$

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Short Answer: Fill in the blank with the appropriate answer.

9. (11 points)

(a) Simplify $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$. _____

(b) $\displaystyle\int x^2 e^{x^3}\, dx =$ _____

(c) $\dfrac{d}{dx}\left(\ln(\sin x)\right) =$ _____

(d) $\dfrac{d}{dx}\left(\sinh^2(x)\right) =$ _____

(e) $\dfrac{d}{dx}\left(e^x + x^3\right) =$ _____

(f) If $f'(x) = e^x + \sin x + x^2$, then $f(x) =$ _____

(g) $\displaystyle\lim_{x\to\infty} \tan^{-1}(x) =$ _____

(h) $\displaystyle\lim_{x\to 2} \frac{x+2}{x^2-4} =$ _____

(i) $\displaystyle\lim_{x\to 0^+} \sin x - \ln x =$ _____

(j) $\displaystyle\int_1^2 4 + 5x\, dx =$ _____

(k) $\dfrac{d}{dx}\left(2^x\right) =$ _____

Free response: Write your answer in the space provided.

10. (8 points)

   (a) If $f(x) = \dfrac{1}{x}$, use the definition of a derivative to set up a limit to find $f'(x)$.

   (b) Find $f'(x)$ by evaluating the limit. (No points will be awarded if differentiation rules are used.)

11. (6 points) Find the dimension of the largest rectangle that can be inscribed between the curve $y = 4 - x^2$ and the x-axis.

12. (4 points) $\lim\limits_{x \to 0} \ln(x) \sin(x)$

13. (3 points) $\dfrac{d}{dx} \left(\ln \left(xe^x - \dfrac{\sin x}{x} \right) \right)$

14. (4 points) $\displaystyle\int \dfrac{x}{x^2 + 4} \, dx$

15. (10 points) Give the following information about the function $f(x) = x^4 - 4x^3$: (If no information is available in a particular category, leave it blank or cross it out. Putting information in where none exists will be treated as an incorrect answer).

All x-intercepts = _____

y-intercept = _____

Intervals for which $f(x)$ is increasing: _____

Intervals for which $f(x)$ is decreasing: _____

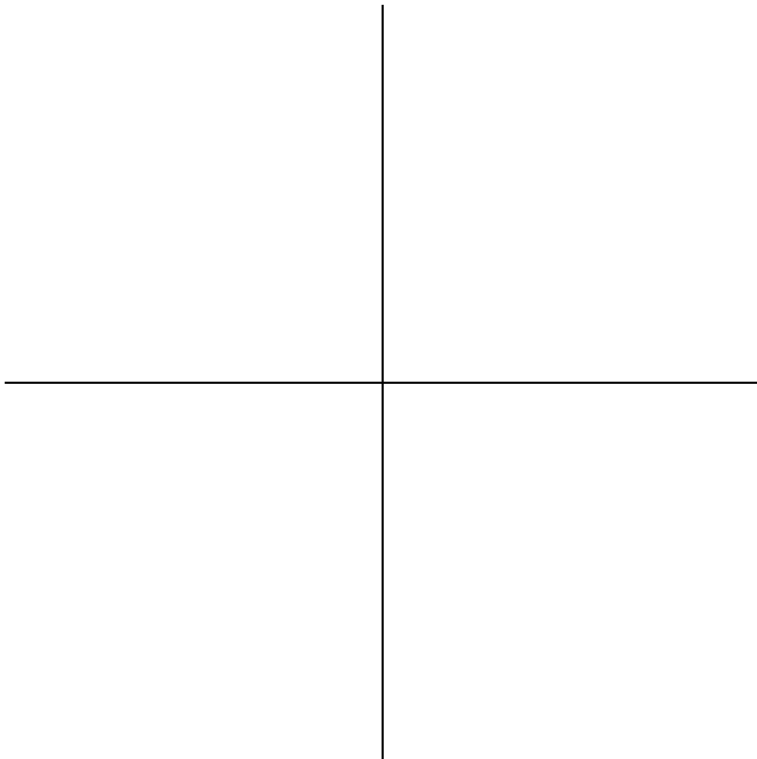Coordinates of all inflection points: _____

Intervals for which $f(x)$ is concave up: _____

Intervals for which $f(x)$ is concave down: _____

Coordinates of any local maximums: _____

Coordinates of any local minimums: _____

Graph the function:

16. (6 points) A certain element has a half life of 20 years. How many years will it take until only 10% of the element remains? $\left(\text{Note: } \ln\left(\frac{1}{2}\right) \approx -.7 \text{ and } \ln\left(\frac{1}{10}\right) \approx -2.3. \text{ You can either leave your answer in terms of logs or give a numerical answer using these approximations.}\right)$

17. (6 points) The equation of the tangent line to the curve $y = \dfrac{1}{x^2}$ at $\left(2, \dfrac{1}{4}\right)$.

18. (6 points) Use linear approximation to estimate $\sqrt{63}$:

19. (6 points) A pump is blowing up a spherical balloon with a pump rate of $10\text{cm}^3/\text{sec}$. How fast is the diameter of the balloon growing when the balloon has a 5cm radius? $\left(\text{Volume of a sphere is given by } \frac{4}{3}\pi r^3.\right)$

20. (6 points) A particle is moving with the given data. Find the position function of the particle.

$$a(t) = \sin(t) + 3\cos(t), \quad s(0) = 0, \quad v(0) = 2.$$