Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 112 (Calculus I)
Final Exam Form A

April 14, 2012, 11:00 a.m. – 2:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 16 to 24, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned
MC	45	
16	10	
17	5	
18	5	
19	6	
Sub	71	

#	Possible	Earned
20	6	
21	6	
22	6	
23	5	
24	6	
Sub	29	
Total	100	

Part I: Multiple Choice. Enter your answer on the scantron. Work will not be collected or reviewed.

1. Find $\lim_{x\to 2}\left(\frac{x^2-2x}{x-2}\right)$.

   a) 1 b) Does not exist c) -2

   d) ∞ e) -1 f) 2

   g) 0 h) $-\infty$ i) None of the above.

2. If for all x you know that $2x^2+x-2 \le f(x) \le 4x^4+2x^2+x-2$, do you have enough information to find $\lim_{x\to 0} f(x)$? If so, what is $\lim_{x\to 0} f(x)$?

   a) Yes, -2 b) Yes, 0

   c) Yes, -1 d) Yes, 2

   e) Yes, 1 f) No, not enough information.

   g) Yes, but none of the above numbers.

3. Find $\lim_{x\to\infty} \frac{5-3x^3}{\sqrt{81x^6-16}}$.

   a) Does not exist b) $-\infty$ c) -3

   d) -1 e) $-\frac{1}{3}$ f) 0

   g) $\frac{1}{3}$ h) 1 i) 3

4. If a function f is defined and twice differentiable on $(-\infty,\infty)$, $f'(2)=0$, and $f''(2)=4$, then

   a) f has an inflection point at $x=2$.

   b) f is increasing in a neighborhood around $x=2$.

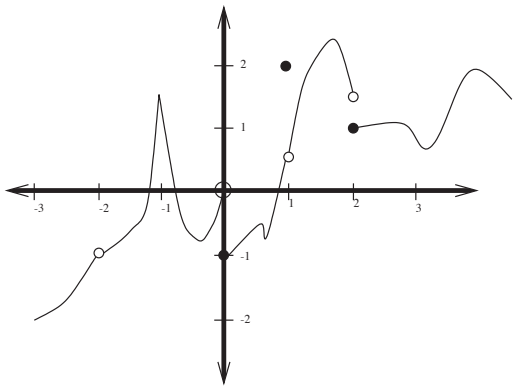
   c) f has a local minimum at $x=2$.

   d) f has a local maximum at $x=2$.

   e) f is decreasing in a neighborhood around $x=2$.

   f) we don't have enough information to prove that any of these are true.

5. Below is the graph of a function. At which of the following points is it continuous?



a) $x = -1$

b) $x = -2$

c) $x = 2$

d) $x = -1$ and $x = -2$

e) $x = 1$

f) $x = 0$

g) f is not continuous at any of these points.

h) f is continuous at all of these points.

6. Find $f'(x)$ where $f(x) = (x^3 + 5x + 11)^7$.

a) $7(x^3+5x+11)^6(3x^2+5)$

b) $7(x^3 + 5x + 11)^6$

c) $(x^3 + 5x + 11)^7$

d) $(3x^2 + 5)$

e) $7(3x^2 + 5)^6$

f) None of the above.

7. Let $f(x) = 3x^5 + 5x^4 + 7$. On which of the following intervals is f increasing?

a) $(-4/3, 0)$

b) $(-1, 0)$

c) $(-\infty, -1)$ and $(0, \infty)$

d) $(-1, \infty)$

e) $(-\infty, \infty)$

f) $(-\infty, -4/3)$ and $(0, \infty)$

g) None of these.

8. What is the maximum y–value of the graph of $f(x) = 4x^2 - x^4 + 1$ on the interval $[-2, 2]$?

a) $y = 2$

b) $y = 9$

c) $y = 5$

d) $y = 6$

e) $y = 0$

f) $y = 4$

g) $y = 1$

h) $y = 3$

i) None of these.

9. Let $k(x) = \sqrt{x-1}$. For what value of c does $k(x)$ satisfy the Mean Value theorem on the interval $[1, 5]$? (In other words, what value of c satisfies $k'(c) = \dfrac{k(5) - k(1)}{5 - 1}$)?

a) 1

b) 2

c) 3

d) 4

e) 5

f) 6

10. Let $h(x) = f(g(x))$, and let $g(2) = 1$, $g'(2) = 2$, $f(1) = 3$, $f'(1) = 5$, $f(2) = 3$, and $f'(2) = 7$. Find $h'(2)$.

a) 14 b) 7 c) 15

d) 2 e) 21 f) 5

g) 10 h) 28 i) 35

j) None of the above.

11. Find the derivative $g'(x)$ of the function $g(x) = x^2 \cos x$.

a) $-2x \sin x$ b) $-\sin 2x$ c) $-2x^3 \sin x \cos x$

d) $2x \sin x$ e) $2x \cos x - x^2 \sin x$ f) $2x \sin x + x^2 \cos x$

g) $\cos 2x$ h) None of these.

12. Find $\lim_{x \to 2} \left(\dfrac{|x-2|}{x-2} \right)$.

a) 1 b) Does not exist c) -1

d) $-\infty$ e) -2 f) ∞

g) 0 h) 2

13. Find an antiderivative of $f(x) = 3x^2 + \dfrac{2}{x^2}$.

a) $x^3 + \dfrac{1}{x}$ b) $x^2 + \dfrac{2}{x^2}$ c) $x^3 - \dfrac{4}{x^3}$

d) $x^3 + \dfrac{4}{x^3}$ e) $x^3 - \dfrac{2}{x}$ f) $x^3 + \dfrac{2}{x}$

14. Find $\dfrac{dy}{dx}$ where $xy = \cos y$.

a) $-\dfrac{y}{(x + \sin y)}$ b) $-\sin y$ c) $-\dfrac{\sin y + y}{x}$

d) $\dfrac{\cos y}{x}$ e) $-\dfrac{x \sin y + \cos y}{x^2}$ f) None of the above.

15. Use linear approximation or differentials to estimate $\sqrt[3]{1000.03}$.

a) 10 b) 10.1 c) 10.01

d) 10.001 e) 10.0001 f) None of the above.

Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

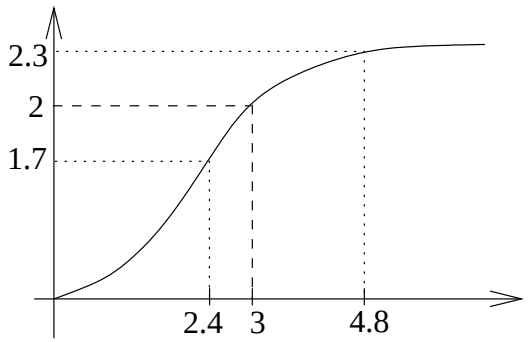
16. (10 points) **Short answer.** Two points each part. You do not need to show your work on this problem.

(a) Find $\lim_{x \to 2^+} \dfrac{1}{x-2}$.

Answer: _____

(b) Use the given graph of f to find the largest number δ such that

$$\text{if} \quad 0 < |x-3| < \delta \quad \text{then} \quad |f(x)-2| < 0.3.$$



Answer: _____

(c) Find the derivative of $\ln(x-3)$.

Answer: _____

(d) Integrate $\displaystyle\int \tan x \sec x \, dx$.

Answer: _____

(e) If $\displaystyle\int_1^5 f(x)\,dx = 12$ and $\displaystyle\int_1^4 f(x)\,dx = 4$, what is $\displaystyle\int_4^5 f(x)\,dx$?

Answer: _____

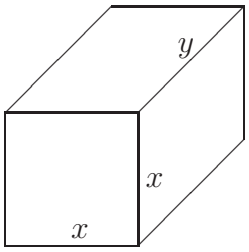
17. (5 points) Use the definition of the derivative to show:

$$\text{If } f(x) = 3 - 2x^2, \text{ then } f'(x) = -4x.$$

No credit will be given if a method besides the definition of the derivative is used.

18. (5 points) A snowball is rolling down a hill in such a way that the radius increases steadily by 2 inches every minute. At what rate is its volume increasing when the radius is 10 inches? (Hint: the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$).

19. (6 points) Suppose that a post office can accept a package for mailing only if the sum of its length and its girth (the perimeter of its cross section) is at most 120 in. What is the maximum volume of a rectangular box with square cross section that can be mailed?



20. (6 points) Find the derivatives.

(a) Find $f'(x)$ if $f(x) = 7^{x^2}$.

(b) Find $g'(x)$ if $g(x) = \displaystyle\int_x^{\ln(x)} \frac{1}{2+t^3} dt$.

21. (6 points) Find the limits.

(a) $\displaystyle\lim_{x\to 0} \frac{\tan x}{\sqrt{x}}$

(b) $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{2i}{n} + 6 \right)$

22. (6 points) Integrate.

(a) $\int_{-2}^{2} (3x+1)^2 \, dx$

(b) $\int \frac{(3-2x)}{(x^2-3x)^{1/3}} \, dx$

23. (5 points) Consider the integral $\int_0^3 (2x+2) \, dx$. Write a Riemann sum approximating the above integral by dividing the interval of integration into n equal parts, and evaluating the function at the right endpoints of the subintervals.

24. (6 points) In this problem, you will analyze the curve given by

$$f(x) = x^4 - 8x^3 + 18x^2 - 8x + 5.$$

(a) Find all intervals where $f(x)$ is concave up and all intervals where $f(x)$ is concave down.

(b) At which values of x does $f(x)$ have an inflection point?

END OF EXAM