Name________________________________

Student Number________________________

Section Number________________________

Instructor_____________________________

Math 113 – Winter 2005

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of 13 short answer questions.
- Problems 2 through 9 are multiple choice questions.
- For problems 10 through 18 give the best answer and *justify* it with suitable reasons and/or relevant work.
- Work on scratch paper will not be graded.
- Do not show your work for problem 1.
- Please write neatly.
- Notes, books, and calculators are not allowed.
- Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

1	/13
M.C.	/24
10	/7
11	/7
12	/7
13	/7
14	/7
15	/7
16	/7
17	/7
18	/7
Total	/100

Math 113 – Winter 2005

Departmental Final Exam

PART I: SHORT ANSWER AND MULTIPLE CHOICE QUESTIONS

Do not show your work for problem 1.

1. Fill in the blanks with the correct answer.

   (a) The integral $\int_0^{\pi/2} \cos(2x)\,dx$ equals ________________________

   (b) The integral $\int \sin x \cos^2 x\,dx$ equals ________________________

   (c) The integral $\int_0^{\infty} \frac{dx}{1+x^2}$ equals ______________

   (d) The radius of convergence of $\sum_{n=0}^{\infty} 2^n x^n$ is ______________

   (e) The first three lowest order terms of the power series of $(1+x)^{1/2}$ may be written as

   ____

   (f) For what values of p does the following improper integral converge?

   $\int_1^{\infty} \frac{dx}{x^p}$ __________________________________

   (g) Indicate which convergence test one could use to determine the convergence/ divergence of

   i. $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ ____

   ii. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ ____

   iii. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ ____

   (h) State the nth term of the MacLaurin series for

   i. e^x ____

   ii. $\frac{1}{1-x}$ ____

   (i) Express in terms of a quotient of integrals the y coordinate of the centroid of the region below $y = f(x)$ with $f(x) > 0$ for all x over $[-2, 2]$.

   ____

   (j) A focus of the hyperbola $\frac{(x-2)^2}{1} - \frac{(y-1)^2}{3} = 1$ is ____________________

Problems 2 through 9 are multiple choice. Each multiple choice problem is worth 3 points. In the grid below fill in the square corresponding to each correct answer.

2	A	B	C	D	E	F	G	H	I
3	A	B	C	D	E	F	G	H	I
4	A	B	C	D	E	F	G	H	I
5	A	B	C	D	E	F	G	H	I
6	A	B	C	D	E	F	G	H	I
7	A	B	C	D	E	F	G	H	I
8	A	B	C	D	E	F	G	H	I
9	A	B	C	D	E	F	G	H	I

2. Find $\displaystyle\int_1^2 \frac{6+x^2+x}{(2+x)(4+x^2)}\,dx$

(a) $2\ln 2 + \arctan 3 - \ln 3 + \frac{1}{2}\arctan 7 - \frac{1}{2}\pi$

(b) $\ln 2 + \frac{1}{8}\pi - \frac{1}{2}\arctan\frac{1}{2}$

(c) $3\ln 2 + \frac{1}{8}\pi - \ln 3 - \frac{1}{2}\arctan\frac{1}{2}$

(d) $2\ln 2 + \arctan 2 - \ln 3 - \frac{1}{4}\pi$

(e) $2\ln 2 + \frac{1}{8}\pi - \ln 3 - \frac{1}{2}\arctan\frac{1}{2}$

(f) 0

(g) π

(h) None of the above

3. The base of a solid is an elliptical region on the xy-plane enclosed by $\dfrac{1}{9}x^2 + \dfrac{1}{4}y^2 = 1$, and cross sections perpendicular to the y axis are squares. Find the volume of the solid.

(a) 20

(b) 64

(c) 12

(d) 96

(e) 30

(f) 48

(g) 18

(h) 72

(i) None of the above

4. Find the length of the graph of $y = x^{1/2} - \dfrac{x^{3/2}}{3}$ for $x \in [1, 6]$.

(a) $\sqrt{7} + 2\sqrt{6} - \frac{1}{3}$

(b) $\sqrt{6} + \frac{7}{3}\sqrt{7} - \frac{2}{3}$

(c) $\sqrt{6} + \frac{7}{3}\sqrt{7} - \frac{5}{3}$

(d) $\sqrt{6} + \frac{7}{3}\sqrt{7} - \frac{4}{3}$

(e) $3\sqrt{6} - \frac{4}{3}$

(f) $\sqrt{7} + \frac{16}{3}\sqrt{2} - \frac{4}{3}$

(g) $\sqrt{6} + \frac{85}{3}$

(h) None of the above

5. Find $\displaystyle\lim_{x\to 0} \frac{2 - x^2 - 2\cos x}{x^4}$

Hint: You might want to consider using power series to do this.

(a) $-\frac{1}{14}$

(b) $-\frac{1}{6}$

(c) $-\frac{1}{3}$

(d) $\frac{1}{6}$

(e) $-\frac{1}{4}$

(f) $-\frac{1}{12}$

(g) $\frac{1}{14}$

(h) $-\frac{1}{10}$

(i) None of the above

6. $\int_3^\infty e^{-t} \sin(4t)\, dt$

(a) $\frac{1}{17}e^{-3}(4\cos 12 + \sin 12)$

(b) $\frac{1}{17}e^{-3}(4\cos 12 - 1 + \sin 12)$

(c) $\frac{1}{17}e^{-3}(4\cos 12 - 3 + \sin 12)$

(d) $\frac{1}{17}e^{-3}(4\cos 12 - 2 + \sin 12)$

(e) $\frac{1}{16}e^{-3}(4\cos 12 + \sin 12)$

(f) $\frac{1}{17}e^{-3}(4\cos 12 + \sin 12)$

(g) The integral does not converge.

(h) None of the above

7. Find the power series expansion for the function $\sin^{-1} x$ or $\arcsin x$ expanded about 0.

Hint: You might want to write the function in the form $\int_0^x$ something dt.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$

(b) $\displaystyle\sum_{k=0}^{\infty} \binom{-1/2}{k} \frac{(-1)^k}{2k+1} x^{2k+1}$

(c) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1}$

(d) $\displaystyle\sum_{k=0}^{\infty} \binom{1/2}{k} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

(e) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!} x^{k+1}$

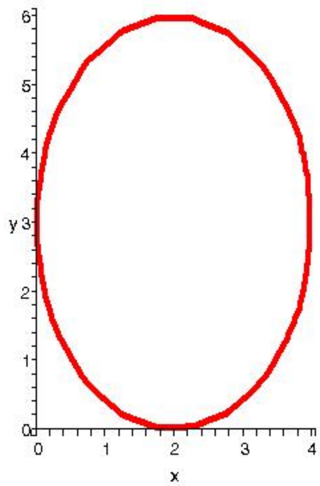
(f) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$

(g) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{k!}$

(h) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

(i) None of the above

8. Identify the equation that best goes with the following graph in rectangular coordinates.



(a) $\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1$

(b) $\frac{(y-3)^2}{9} - \frac{(x-2)^2}{4} = 1$

(c) $x = y^2 - 2$

(d) $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$

(e) $y = 4 - x^2$

(f) $\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1$

(g) $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} = 1$

(h) $\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$

(i) None of the above

9. Which of the following integrals represents the surface area of the surface generated by revolving the curve $y = e^{2x}$, $0 \le x \le 1$, about the line $y = -2$.

(a) $\int_0^1 2\pi(e^{2x} - 2)\sqrt{1 + 4e^{4x}}\,dx$

(b) $\int_0^1 2\pi(e^{2x} + 2)\sqrt{1 + 4e^{4x}}\,dx$

(c) $\int_0^1 2\pi(e^{2x})\sqrt{1 + 4e^{4x}}\,dx$

(d) $\int_0^1 2\pi(e^{2x})\sqrt{1 + e^{4x}}\,dx$

(e) $\int_0^1 2\pi(e^{2x} - 2)\sqrt{1 + e^{4x}}\,dx$

(f) $\int_0^1 2\pi(e^{2x} - 2)\,dx$

(g) $\int_0^1 2\pi(e^{2x} + 2)\,dx$

(h) $\int_0^1 2\pi(e^{2x})\,dx$

(i) None of the above

The answers to the multiple choice MUST be entered on the grid on the previous page. Otherwise, you will not receive credit.

Part II: Written Solutions

For problems 10 – 18, write your answers in the space provided. Neatly show your work for full credit.

10. Find a formula for $\int \sqrt{b^2 - a^2x^2}\, dx$. Here a, b are positive constants.

11. Find the area of the region bounded by the curve $x = y - y^2$ and the line $y = -x$.

12. Find the volume of the solid generated by revolving the region enclosed by $y = 4$ and $y = 3(x-3)^2 + 1$ about the y-axis.

13. Determine the values of p for which the integral $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^p}dx$ converges. Justify your answer.

14. (a) Find a Maclaurin series which represents the function $\dfrac{\sin\sqrt{x}}{\sqrt{x}}$ when $x > 0$.

(b) Hence calculate $\lim\limits_{x \to 0+} \dfrac{\sin\sqrt{x}}{\sqrt{x}}$.

(c) Find the interval of convergence of this power series.

15. Find the Taylor polynomial of degree 3 for $f(x) = 4x^3 + 3x^2 + 2x + 1$ which is centered at 1.

16. Compute $I_1 = \int x \ln x \, dx$ and determine a reduction formula for

$$I_n = \int x (\ln x)^n \, dx, \quad n > 1.$$

17. Sketch the closed curve $r = 7 \cos\left(\theta - \frac{\pi}{4}\right)$ and determine the area enclosed by the curve.

18. (a) For which values of x does $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k} (1 - e^x)^k$ converge?

(b) What is the sum of this series?

—End—