Name________________________________

Student Number________________________

Section Number________________________

Instructor____________________________

Math 113 – Fall 2005

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of 13 short answer questions.
- Problem 2 consists of 3 T/F questions.
- Problems 3 through 9 are multiple choice questions.
- For problems 10 through 18 give the best answer and *justify* it with suitable reasons and/or relevant work.
- Work on scratch paper will not be graded. Do not show your work for problem 1 through 9.
- Please write neatly.
- Notes, books, and calculators are not allowed.
- Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

1	/13
2	/3
M.C.	/21
10	/7
11	/7
12	/7
13	/7
14	/7
15	/4
16	/10
17	/7
18	/7
Total	/100

Math 113 – Fall 2005

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Part I: Short Answer and Multiple Choice Questions

Do not show your work for problems in this part.

1. Fill in the blanks with the correct answer.

   (a) The integral $\int \cos(x+2)\,dx$ equals ____________________

   (b) The integral $\int \sec x \tan x\,dx$ equals ____________________

   (c) The integral $\int_0^1 \frac{dx}{1+x^2}$ equals ____________________

   (d) The integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ equals ____________________

   (e) The integral $\int \tan^2 x\,dx$ equals ____________________

   (f) The integral $\int_0^1 \frac{dx}{\sqrt{x}}$ equals ____________________

   (g) The integral $\int_0^\infty \frac{dx}{x^3}$ equals ____________________

   (h) The integral $\int \frac{x}{\sqrt{1+x^2}}\,dx$ equals ____________________

   (i) Give the limit of the sequence $\left\{\left(1-\frac{1}{n}\right)^n\right\}$ as $n \to \infty$ if it is convergent, otherwise write DIVERGENT.

   ____

   (j) State the integration by parts formula:

   ____

   (k) Give a limit definition of the improper integral $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$

   ____

   (l) Let State the $(2n)$-th term of the MacLaurin series for $\frac{\sin x}{x}$

   ____

(m) The integral $\int \cot x \, dx$ equals ______________________________

2. True/False: Write T if statement always holds, F otherwise.
Let $\sum a_n = \sum_{n=1}^{\infty} a_n$ be an arbitrary series.

(a) _____ If $\{a_n\}$ is a positive decreasing sequence then $\sum (-1)^n a_n$ converges

(b) _____ If $\sum a_n$ converges then $a_n \to 0$

(c) _____ If the partial sums of $\sum a_n$ are bounded, then $\sum a_n$ converges

Problems 3 through 9 are multiple choice. Each multiple choice problem is worth 3 points. In the grid below fill in the square corresponding to each correct answer.

3	A	B	C	D	E	F	G	H	I
4	A	B	C	D	E	F	G	H	I
5	A	B	C	D	E	F	G	H	I
6	A	B	C	D	E	F	G	H	I
7	A	B	C	D	E	F	G	H	I
8	A	B	C	D	E	F	G	H	I
9	A	B	C	D	E	F	G	H	I

3. The most appropriate first step to integrate $\int \frac{x^2 - 1}{3x^3 - x^2} \, dx$ would be

(a) Integration by parts

(b) Partial fractions

(c) Trigonometric Substitution

(d) Other (non trigonometric) substitution

(e) Differentiate the integrand

(f) None of these

4. The series $x^2 + x^4 + \frac{x^6}{2} + \frac{x^8}{6} + \cdots = \sum_{n=0}^{\infty} \frac{x^{(2n+2)}}{n!}$ converges to the function

(a) $\frac{x^2}{1+x^2}$

(b) $x^2 \tan^{-1} x$

(c) e^{x^2+2}

(d) $x^2 e^{x^2}$

(e) $x^2(\sin x^2 + \cos x^2)$

(f) $\sin x^2 + \cos x^2$

(g) None of these

5. The improper integral $\int_0^\infty x e^{-x} dx$ converges to

(a) 0

(b) $1/e$

(c) $1/2$

(d) 1

(e) 2

(f) e

(g) None of these

(h) It doesn't converge

6. The length of the curve $y = \cosh x$ from $x = 0$ to $x = 1$ is

(a) $\sinh 1$

(b) $\cosh 1$

(c) $\cosh^2 1 - \cosh^2 0$

(d) 1

(e) ∞

(f) a real number in (0,1)

(g) Imaginary

(h) None of these

7. The area enclosed by the polar curve $r = 3 + \sin\theta$ is

(a) 5π

(b) 4π

(c) 9π

(d) $\pi/4$

(e) 4.5π

(f) 19π

(g) $9\pi^2$

(h) None of these

8. The interval of convergence of the power series $\sum_{n=1}^{\infty} n^2(5x-3)^n$ is

(a) $(-3/5, 3/5)$

(b) $(-5/3, 5/3)$

(c) $(0, 1)$

(d) $(-1, 1)$

(e) $(2/5, 4/5)$

(f) $(1/5, 1)$

(g) $(0, \infty)$

(h) $(-\infty, \infty)$

(i) None of the above

9. The coefficient of x^3 in the series expansion of $(1+x)^{1/4}$ is

(a) $\frac{1}{4^3} = \frac{1}{64}$

(b) $\frac{1}{4^3 3!} = \frac{1}{384}$

(c) $\frac{6}{4^3 3!} = \frac{1}{64}$

(d) $\frac{15}{4^3 3!} = \frac{5}{128}$

(e) $\frac{20}{4^3 3!} = \frac{5}{96}$

(f) $\frac{21}{4^3 3!} = \frac{7}{128}$

(g) $\frac{25}{4^3 3!} = \frac{25}{384}$

(h) $\frac{35}{4^3 3!} = \frac{35}{384}$

(i) None of the above

The answers to the multiple choice MUST be entered on the grid on the previous page. Otherwise, you will not receive credit.

Part II: Written Solutions

For problems 10 – 18, write your answers in the space provided. Neatly show your work for full credit.

10. (a) Evaluate the integral $\displaystyle\int_0^1 t^2 e^t \, dt$.

(b) Expand in partial fraction form $\dfrac{x^2+3}{x^2-1}$.

(c) Evaluate the integral $\displaystyle\int \frac{x^2+3}{x^2-1}\, dx$.

11. Evaluate the integral $\displaystyle\int \frac{1}{4-3\sin x}\, dx$.

12. The region bounded by $y = x$ and $y = 2x^2$ is revolved about the **y-axis** ; find the volume of the solid generated.

13. Find the area of the surface of revolution generated by revolving the curve $y = \sqrt{x},\ 0 \le x \le 4$, about the x-axis.

14. Find the centroid of the region bounded by the curves

$$y = \sqrt{1+x^2}, \;\; x = 1 \;\; \text{and} \;\; y = 1 + x.$$

Express you answer in terms of unevaluated integrals. (Note: You should simplify the integrands as much as possible.)

15. If a region in the first quadrant, with area 10π and centroid at the point $(1, 12)$, is revolved around the line $x = -5$, find the resulting volume of revolution.

16. Determine whether each infinite series is absolutely convergent, conditionally convergent, or divergent. Give reasons for your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{3n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} (3^{-n} - 5^{-n})$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$

17. (a) Determine the power series expansion of $\int \tan^{-1} x \, dx$.

(b) Find first two nonzero terms of the Taylor series of $\ln(1 + \sin^2 x)$ at $x = \pi$. What is the remainder after these terms?

18. Given the polar curve $r = \theta^2,\ 0 \le \theta \le 3/2$,

   (a) sketch the curve;

   (b) find the area swept out by the curve;

   (c) find the arc length.

—*End*—