Name________________________________

Student Number________________________

Section Number________________________

Instructor_____________________________

Math 113 – Fall 2006

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of 9 short answer questions.
- Problems 2 through 8 are multiple choice questions.
- For problems 9 through 18 give the best answer and *justify* it with suitable reasons and/or relevant work.
- Work on scratch paper will not be graded. Do not show your work for problem 1 through 9.
- Please write neatly.
- Notes, books, and calculators are not allowed.
- Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

1	/9
M.C.	/21
9	/7
10	/7
11	/7
12	/7
13	/7
14	/7
15	/7
16	/7
17	/7
18	/7
Total	/100

Math 113 – Fall 2006

Departmental Final Exam

Part I: Short Answer and Multiple Choice Questions

Do not show your work for problem 1.

1. Fill in the blanks with the correct answer.

   (a) Does the improper integral $\displaystyle\int_0^\infty \frac{dx}{e^x+1}$ converge (yes or no) ____________________

   (b) The integral $\displaystyle\int \frac{\cos x}{\sin^3 x}\,dx$ equals ____________________

   (c) The integral $\displaystyle\int_1^{e^2} \frac{dx}{2x}$ equals ____________________

   (d) $\dfrac{x^2}{4} - \dfrac{y^2}{25} = 1$ is the equation of a/an ____________________

   (e) The radius of convergence of $\displaystyle\sum_{n=0}^{\infty} 3^n x^n$ is ____________________

   (f) If $n > 1$, the integral $\displaystyle\int_1^\infty \frac{dx}{x^n}$ equals ____________________

   (g) The series $x^2 - \dfrac{x^4}{3!} + \dfrac{x^6}{5!} - \dfrac{x^8}{7!} + \ldots$ is the MacLaurin series for the function ____________________

   (h) The integral $\displaystyle\int x\ \sin x\, dx$ equals ____________________

   (i) The series $2 - \dfrac{2}{3} + \dfrac{2}{9} - \dfrac{2}{27} + \ldots$ converges to ____________________

Problems 2 through 8 are multiple choice. Each multiple choice problem is worth 3 points. In the grid below fill in the square corresponding to each correct answer.

2	A	B	C	D	E	F	G	H	I
3	A	B	C	D	E	F	G	H	I
4	A	B	C	D	E	F	G	H	I
5	A	B	C	D	E	F	G	H	I
6	A	B	C	D	E	F	G	H	I
7	A	B	C	D	E	F	G	H	I
8	A	B	C	D	E	F	G	H	I

2. Which of the following integrals represents the surface area of the surface generated by revolving the curve $y = \tan x$, $0 \le x \le \pi/4$, about the line $y = -2$?

   (a) $\int_0^{\pi/4} \pi(\tan x + 2)\sqrt{1+\sec^2 x}\, dx$

   (b) $\int_0^{\pi/4} 2\pi(\tan x + 2)\sqrt{1+\sec^2 x}\, dx$

   (c) $\int_0^{\pi/4} \pi(\tan x + 2)\sqrt{1+\sec^4 x}\, dx$

   (d) $\int_0^{\pi/4} 2\pi(\tan x + 2)\sqrt{1+\sec^4 x}\, dx$

   (e) $\int_0^{\pi/4} \pi(\tan x - 2)\sqrt{1+\sec^4 x}\, dx$

   (f) $\int_0^{\pi/4} 2\pi(\tan x - 2)\sqrt{1+\sec^2 x}\, dx$

   (g) $\int_0^{\pi/4} \pi(\tan x - 2)\sqrt{1+\sec^2 x}\, dx$

   (h) $\int_0^{\pi/4} 2\pi(\tan x - 2)\sqrt{1+\sec^4 x}\, dx$

   (i) None of the above

3. Which of the following substitutions will best simplify the integral $\int \sqrt{3+2x-x^2}\, dx$?

   (a) $x = 1 - 2\sec u$

   (b) $x = \sqrt{3} + 2\cosh u$

   (c) $x = \sqrt{3}\cos u$

   (d) $x = \sqrt{3} - 2\cosh u$

   (e) $x = \sqrt{3}\sin u$

   (f) $x = 1 + 2\sin u$

   (g) $x = 2\sin u$

4. Consider the region R that is the portion of the circle $x^2+y^2 = 1$ that lies in the first quadrant. What is the volume of the solid generated by revolving R about the line $x + y = 2$?

(a) $\dfrac{\pi}{2\sqrt{2}}$ (d) $\dfrac{\pi^2}{2}$ (g) $\dfrac{\pi^2\sqrt{2}}{3}$

(b) $\dfrac{\pi}{2}$ (e) $\dfrac{\pi^2}{3\sqrt{2}}$ (h) $\dfrac{\pi^2}{2\sqrt{2}}$

(c) $\dfrac{\pi\sqrt{2}}{3}$ (f) $\dfrac{\pi^2}{4}$ (i) None of the above

5. The series $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{n!}$ converges to

(a) $\ln 3$ (d) $\dfrac{3^{n+1}}{n+1}$ (g) $\cos 3$

(b) $\ln 2$ (e) ∞ (h) $e^3 - 4$

(c) $\ln(3) - 1$ (f) e^3 (i) 3^e

6. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} n^2(7x-3)^n$ is

(a) $\left(-\dfrac{3}{7}, \dfrac{3}{7}\right)$ (d) $(0, 1)$ (g) $(0, \infty)$

(b) $\left(-\dfrac{7}{3}, \dfrac{7}{3}\right)$ (e) $\left(\dfrac{1}{7}, 1\right)$ (h) $(-\infty, \infty)$

(c) $(-1, 1)$ (f) $\left(\dfrac{2}{7}, \dfrac{4}{7}\right)$ (i) None of these

7. The integral $\displaystyle\int_2^{e+1} (x-1)\ln(x-1)\, dx$ is equal to

(a) $\dfrac{e^2-1}{2}$ (d) $\dfrac{e^2+1}{4}$

(b) $e^2 + 1$ (e) $\dfrac{e^2-1}{4}$

(c) $\dfrac{e^2+1}{2}$ (f) $e^2 - 1$

8. The graph of the polar equation $r = 2\cos(n\theta)$ has how many petals?

(a) n petals if n is even, $2n$ petals if n is odd

(b) $n/2$ petals if n is odd, n petals if n is even

(c) n petals if n is odd, $2n$ petals if n is even

(d) $2n$ petals

(e) n petals

(f) $n/2$ petals

(g) None of these

The answers to the multiple choice MUST be entered on the grid on page 2. Otherwise, you will not receive credit.

Part II: Written Solutions

For problems 9 – 18, write your answers in the space provided. Neatly show your work for full credit.

9. Evaluate each integral

(a) $\displaystyle\int \frac{dx}{2 + x - x^2}$

(b) $\displaystyle\int \sec^3(2x)\,dx$

10. Find the general solution, in the form $y = f(x)$, to the differential equation

$$\frac{dy}{dx} = (4 + y^2)(4 + x^2).$$

11. Find the length of the graph of $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, on the interval $1 \leq x \leq 2$.

12. Find the centroid of the region that lies within the first quadrant and is bounded above by $y = 1 - x^2$.

13. Find the area enclosed by the polar curves $r = 2 - \cos\theta$ and $r = 1$.

14. Use the first three non-zero terms of the MacLaurin series for e^{-x^2} to estimate the definite integral $\displaystyle\int_0^2 e^{-x^2}\,dx$. Write your answer as a fraction, if possible.

15. Find the mass of the circular region $x^2 + y^2 \leq 1$, whose density at each point is twice the distance from the point to the origin.

16. Find the sum of the power series $\sum_{n=1}^{\infty} nx^{n-1}$ (as a rational function of x).

17. Determine whether each of the following infinite series converges. State any convergence/divergence test you used.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^{30}+2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

18. Find the definite integral $\int_0^1 x^3\sqrt{1-x^2}\,dx$.