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Exam 1 Review
Math 118 –All Sections

This exam will cover sections 1.1-1.6 and 2.1-2.3 of the textbook. No books, notes, calculators or other aids are allowed on this exam. There is no time limit. It will consist of 25 multiple choice questions. All exam questions will have the option 'None of the above', but when writing the exam, we intend to always include the correct answer among the other options. This means 'None of the above' will be a correct answer only if there is a typo or similar type mistake.

It is important that you know the terms, notations and formulas in the book. Some exam questions will test definitions, notations, or knowledge of basic facts of equations directly. The questions on this review that test the basics are marked with a B. There are a disproportionate number of basic questions on this review, because almost all problems require the knowledge of this basic information. Most exam questions will mimic homework problems as much as possible, considering the homework is free response and the exam is multiple choice. Finally, up to 20% of exam questions will be what I call extension questions. Extension questions test principles covered for this exam, but the questions may look unfamiliar and may require some ingenuity.

The review questions included are not all possible types of questions. They are just representative of the types of questions that could be asked to test the given concept(s).

The following are the concepts which will be tested on this exam.

A.1 Know the definition of a set, and the terminology and notations associated with sets: element, empty set, set builder notation, subset, proper subset, universal set, complement, intersection, union, disjoint. (1.1)

A.2 Know the subset properties (p.4) and the formula to find the number of subsets of a set (p.5). Be able to perform set operations on given sets. Be able represent sets in various ways, including Venn Diagrams, words, and tables. (1.1)

1. (B) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$. Which of the following statements are correct:

    (a) A and B are disjoint.

    (b) A is a proper subset of B

    (c) $U = A \cup B \cup C$

    (d) $C' = B$

    (e) $B \cap C = \emptyset$ EMPTY SET

    (f) None of the above

2. (B) Let $A = \{x | x$ is an odd number between 4 and 10$\}$. How many subsets of A are there?

    (a) 0
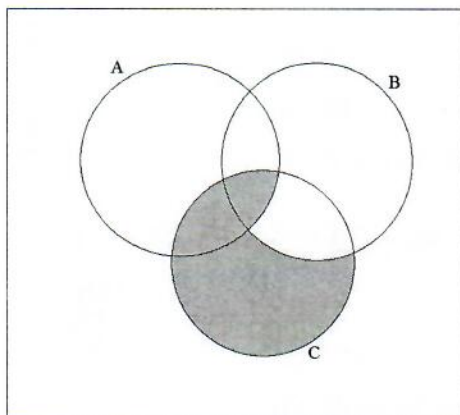
    (b) 2

    (c) 4

    (d) 6

    (e) 8

    (f) None of the Above

$$\cap = (5, 7, 9) = 3$$

$$2^3 = 8$$

3. Which of the sets below corresponds to the shaded area of the given Venn Diagram?

U = OR
$\cap$ = and

(a) $A \cap B'$

(b) $(A \cup B') \cap C$

(c) $(A' \cap B') \cap C$

(d) $A \cap (B' \cap C)$

(e) $B' \cap C$

(f) None of the Above

4. Let M be married people, let T be people over 30, and let S be BYU students. Then $T' \cap (S \cap M)$ can be described as:

(a) The set of BYU students who are married and over 30.

(b) The set of all married people who go to BYU or who are over 30

(c) The set of married BYU students age 30 and younger.

(d) The set of all people who are 30 and under and are either married, or are a BYU student.

(e) The set of all people who are married and a BYU student, or who are 30 and under.

(f) None of the Above

5. (B) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be a universal set. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$. Find $(A \cup B)' \cap C$.
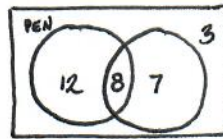
(a) $\{1\}$

$(A \cup B) = \{1, 2, 3, 4, 5, 6, 8\}'$

(b) $\{1, 3\}$

(c) $\{1, 3, 5, 9\}$ $\{7, 9\} \cap \{1, 3, 5, 7, 9\}$

(d) $\{7\}$

(e) $\{7, 9\}$

(f) None of the Above

A.3 Given information about the sizes of certain sets, know how to find the sizes of other sets. In particular, you should know how to use the union rule for sets (p.14) and be able to use Venn Diagrams and/or tables to determine the sizes of sets. (1.2)

UNION RULE: $A + B - A \cap B = A \cup B$

6. In classroom of 30 students, 15 students take notes in pencil, 20 students take notes in pen, and 8 students take notes in pen and pencil. How many students students do not use a pen or pencil to take notes?
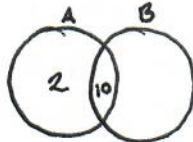
(a) 2
(b) 3
(c) 5

PEN
12 (8) 7   3

(d) 8
(e) 13
(f) None of the Above

7. Let U be the universal set and $n(U) = 50$. If $n(A' \cup B') = 40$, $n(A) = 12$, find $n(A \cap B')$.
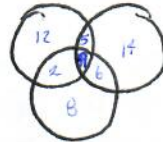
(a) 2
(b) 3
(c) 5

A   B
2 (10)

50
-40
10

(d) 8
(e) 13
(f) None of the Above

8. If $n(A) = 28$, $n(B) = 34$, $n(C) = 25$, $n(A \cap B) = 14$, $n(B \cap C) = 15$, $n(A \cap C) = 11$, $n(A \cap B \cap C) = 9$, find the number of elements in $(A \cup B)' \cap C$.

(a) 2
(b) 3
(c) 5

12   5   17
2   9   6
8

(d) 8
(e) 13
(f) None of the Above

9. Using the information of the table, find $n((S \cup O)' \cup M)$.

	Undecided Major (U)	Pre-Management (M)	Other (O)	Total
Freshman(F)	20	15	5	40
Sophomore(S)	20	40	20	80
Junior(J)	10	45	5	60
Senior (R)	4	10	6	20
Total	54	110	36	200

$(S \cup O) = 96$
$(S \cup O)' = 104$
$(M) = 110$

$- 96$
104

DOUBLE = $15 + 45 + 10 = 70$

$104 + 110 - 70 = 144$

(a) 90
(b) 166
(c) 135

(d) 144
(e) 160
(f) None of the Above

A.4 Know the definition of probability and the terminology and notations associated with probability: experiment, trial, sample space, event, simple event, certain event, impossible event, mutually exclusive, empirical probability. (1.3)

SIMPLE = ONLY ONE POSSIBLE OUTCOME
CERTAIN = SAMPLE SPACE
IMPOSSIBLE = ∅

A.5 Know the basic facts for probabilities (1.3/1.4):

(i) For any event E, $0 \leq P(E) \leq 1$.

(ii) If S is a sample space with n distinct outcomes $s_1, s_2, \ldots s_n$, and for each outcome s_i, the probability of s_i is p_i, then $p_1 + p_2 + \cdots p_n = 1$. Or, in other words, if S is our sample space, then $P(S) = 1$.

10. (B) An experiment has a possible sample space $S = \{s_1, s_2, s_3, s_4, s_5\}$. If $P(s_1) = \frac{1}{2}$, $P(s_2) = \frac{1}{6}$, $P(s_3) = \frac{1}{4}$, and $P(s_4) = \frac{1}{6}$, what can be said about $P(s_5)$?

(a) $P(s_5) = 1/6$

(b) $P(s_5) = 0$

(c) $P(s_5) = -\frac{1}{12}$

(d) There is not enough information to determine $P(s_5)$.

(e) This is not a valid assignment of probabilities.

(f) None of the Above

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{4} + \frac{1}{6} + s_5 = 1$$

$$\frac{12}{24} + \frac{4}{24} + \frac{6}{24} + \frac{4}{24} + s_5 = 1$$

$$\frac{26}{24} + s_5 = 1$$

11. (B) An experiment consists of choosing a letter and then choosing a single digit number. What is the size of the sample space?

(a) 26

(b) 36

(c) 234

$26^1 \times 10^1 = 260$

(d) 260

(e) 26^{10}

(f) None of the Above

12. (B) A box contains 4 red marbles, 3 blue marbles, and 1 white marble. The experiment is to draw one marble and flip a coin. If A is the event that a head is flipped, which of the events below is mutually exclusive to A?

(a) Flip a head

(b) Flip a tail

(c) Pick a red marble

$\overset{A}{\overset{}{\underset{A'}{\Big\langle}}}$

(d) Pick a blue marble

(e) Pick a white marble

(f) None of the Above

A.6 Know when you can use the Basic Probability Principle and know how to use it (p.23). Be able to compute probabilities using the basic probability principle, tables, tree diagrams, or other methods. In particular, you should understand the terminology associated with a standard deck of cards, which for us will be 52 cards in four suits as described on page 24. An ace is NOT a face card. (1.3/1.4)

A.7 Know the Union Rule for Probability (p.29) and be able to use it. Know and be able to use the complement rule (p.30). (1.4)

13. (B) If $P(A) = 0.7$, $P(B) = 0.5$, and $P(A \cap B) = 0.4$, find $P(A \cup B)$.

(a) 0.2

(b) 0.3

(c) 0.5

(d) 0.7

(e) 0.8

(f) None of the Above

$$A + B - A \cap B =$$
$$.7 + .5 - .4 = .8$$

14. Using the information of the table, find $P(J \cup M)$.

	Undecided Major (U)	Pre-Management (M)	Other (O)	Total
Freshman(F)	20	15	5	40
Sophomore(S)	20	40	20	80
Junior(J)	10	45	5	60
Senior (R)	4	10	6	20
Total	54	110	36	200

JUM = J + M - J∩M

= 60 + 110 - 45

= 125

$\frac{6}{8}$; $\frac{125}{200}$; $\frac{5}{8}$

(a) $\frac{1}{6}$

(b) $\frac{2}{5}$

(c) $\frac{4}{7}$

(d) $\frac{5}{8}$

(e) $\frac{3}{4}$

(f) None of the Above

15. Two fair, six sided dice are rolled. What is the probability that they show different numbers?

(a) $\frac{1}{2}$

(b) $\frac{5}{9}$

(c) $\frac{2}{3}$

1 - P(Same number)

: 1 - $\frac{6}{36}$

: 1 - $\frac{1}{6}$ = $\frac{5}{6}$

(d) $\frac{25}{36}$

(e) $\frac{5}{6}$

(f) None of the Above

16. There are three jars. The first is picked 1/2 of the time, the second is picked 1/4 of the time, and the third is picked 1/4 of the time. The first jar contains 1 white marble and 3 black marbles. The second jar contains 2 white marbles and 2 black marbles. The third jar contains 3 white marbles and one black marble. If a jar is chosen, then a marble is chosen from that jar, what is the probability that a white marble is chosen?

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{5}{8}$

(e) $\frac{7}{16}$

(f) None of the Above

$(\frac{1}{4})(\frac{1}{2}) = \frac{1}{8} = \frac{2}{16}$

$(\frac{1}{4})(\frac{1}{2}) = + \frac{1}{8} = \frac{2}{16}$

$(\frac{1}{4})(\frac{3}{4}) = \frac{3}{16}$

$\frac{7}{16}$

A.8 Given the probability of an event E, find the odds in favor of E. Given the odds in favor of an event E, find the probability of E. (1.4)

17. (B) If the odds in favor of E are $3 : 7$, find $P(E)$.

(a) $\frac{1}{3}$

(b) $\frac{1}{7}$

(c) $\frac{3}{7}$

$\frac{3}{3+7}$: $\frac{3}{10}$

(d) $\frac{3}{10}$

(e) $\frac{7}{10}$

(f) None of the Above

A.9 Know the definition of conditional probability (p.41) and the product rule of probability (p.43). Be able to use these to solve for $P(E|F)$ or $P(E \cap F)$ as required. (1.5)

A.10 Know the definition of independent events and the product rule for independent events (p.47). (1.5)

INDEPENDENT RULE : $P(E \cap F) = P(E) \cdot P(F)$

18. (B) Using the information of the table, find $P(U|F)$.

	Undecided Major (U)	Pre-Management (M)	Other (O)	Total
Freshman(F)	20	15	5	40
Sophomore(S)	20	40	20	80
Junior(J)	10	45	5	60
Senior (R)	4	10	6	20
Total	54	110	36	200

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

P $\quad \frac{20}{40} : \frac{1}{2}$

(d) $\frac{1}{5}$

(e) $\frac{2}{5}$

(f) None of the Above

19. Use the given Venn Diagram to find $P(A|B)$.

$\dfrac{P(A|B)}{P(B)} = \dfrac{.2}{.5}$

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $\frac{1}{5}$

(e) $\frac{2}{5}$

(f) None of the Above

20. In a certain community, 36 percent of families own a dog, and 22 percent of families that own a dog also own a cat. In addition, 30 percent of families own a cat. What is the probability that a randomly selected family owns both a dog and a cat?

(a) .06125

(b) .0660

(c) .0792

$(.36)(.22) = .0792$

(d) .08125

(e) .0847

(f) None of the Above

21. (B) A box contains 4 red marbles, 3 blue marbles, and 1 while marble. The experiment is to draw one marble and flip a coin. If A is the event that a red marble is drawn, which of the events below is <u>independent</u> from A?

    (a) Flip a head   (d) Pick a blue marble and flip a head

    (b) Pick a blue marble   (e) All of the above are independent from A

    (c) Pick a white marble   (f) None of the Above

22. Let A and B be independent events. If $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ and $P(A \cup B)$.

    (a) $P(A \cap B) = \frac{1}{15}$; $P(A \cup B) = \frac{7}{15}$ (d) $P(A \cap B) = \frac{2}{15}$; $P(A \cup B) = \frac{7}{15}$ $A \cap B = A \cdot B = \frac{1}{3} \cdot \frac{1}{5} = \frac{1}{15}$

    (b) $P(A \cap B) = \frac{2}{15}$; $P(A \cup B) = \frac{8}{15}$ (e) $P(A \cap B) = \frac{7}{15}$; $P(A \cup B) = \frac{8}{15}$

    (c) $P(A \cap B) = \frac{1}{15}$; $P(A \cup B) = \frac{8}{15}$ (f) None of the Above

A.11 Know Bayes' Theorem (p. 56), including the special case for Bayes' Theorem (p. 54). Know how to use Bayes' Theorem, including using tree diagrams and table information as necessary. (1.6)

23. Suppose that 5 percent of men and .25 percent of women are colorblind. At a certain university, 40 percent of the students are female. Find the probability that a colorblind student at that university is male.

    (a) $\frac{29}{30}$

    (b) $\frac{30}{31}$

    (c) $\frac{31}{32}$

    (d) $\frac{32}{33}$

    (e) $\frac{33}{34}$

    (f) None of the Above

$$\frac{(.6)(.05)}{(.6)(.05) + (.4)(.0025)} = \frac{.030}{.031} = \frac{30}{31}$$

[Tree diagram: .6 to M, .05 to CB, .95 to CB'; .4 to F, .0025 to CB, .9975 to CB']

24. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals, and 24 percent as Conservatives. In a recent election, 30 percent of the Independents, 60 percent of the Liberals, and 50 percent of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is an Independent?

    (a) $\frac{23}{27}$

    (b) $\frac{23}{39}$

    (c) $\frac{23}{51}$

    (d) $\frac{23}{73}$

    (e) $\frac{23}{95}$

    (f) None of the Above

	% OF CITY		% OF VOTED		
I	46 %	×	30%	=	.138
L	30 %	×	60%	=	.180
C	24%	×	50%	=	.120

$$\frac{.138}{.438} = \frac{69}{219} = \frac{23}{73}$$

.138 + .180 + .120 = .438

25. The entirely fictionalized table below gives the proportion of U.S. adults in each age group, as well as the proportion of each group that eats peanuts. Find the probability that a randomly selected adult who does NOT eat peanuts is between 45 and 64.

Age	Proportion Of Population	Proportion that Eats Peanuts	Proportion that Does NOT Eat Peanuts
18-44	0.50	0.50	.50
45-64	0.35	0.60	.40
65-74	0.10	0.90	.10
75 years and older	0.05	0.40	.60

(a) $\frac{14}{43}$

(b) $\frac{9}{14}$

(c) $\frac{7}{19}$

(d) $\frac{3}{23}$

(e) $\frac{7}{57}$

(f) None of the Above

$= \dfrac{(.35)(.4)}{(.35)(.4) + (.50)(.50) + (.10)(.10) + (.05)(.60)}$

$= \dfrac{.14}{.14 + .25 + .01 + .03} = \dfrac{.14}{.43}$

A.12 Know the multiplication principle and the terminology and notations associated with counting: permutation, factorial, distinguishable permutation, combination. (2.1/2.2)

A.13 Know how to determine between a permutation and a combination. Be able to calculate permutations, distinguishable permutations, and combinations and answer other basic counting problems. (2.1/2.2)

26. (B) Find 6!

(a) 35

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

(b) 720

(c) 840

(d) 1050

(e) 1260

(f) None of the Above

27. (B) Find $C(7, 4)$

(a) 35

$\dfrac{n!}{r!\,(n-r)!} = \dfrac{7}{4!\ 3!}$

(b) 720

(c) 840

(d) 1050

(e) 1260

(f) None of the Above

28. (B) Find $P(n, n-1)$

(a) 0

(b) 1

$\dfrac{n!}{(n-r)!}$

(c) n

(d) $n - 1$

(e) $n!$

(f) None of the Above

$= \dfrac{n!}{(n-(n-1))!} = \dfrac{n!}{1!} = n!$

29. In how many ways can the letters of the word 'propose' be arranged?

(a) 35

(b) 720

(c) 840

$$\frac{7!}{2!\,1!\,2!\,1!\,1!} = \frac{7!}{2!\,2!} = \frac{5040}{4}$$

(d) 1050

(e) 1260

(f) None of the Above

30. Camille has 8 friends and Don has 6 friends. In how many ways can they invite six friends to a party if Camille will invite four friends and Don will invite two?

(a) 144

(b) 320

(c) 336

$$_8C_4 \times {_6}C_2 = 1050$$

(d) 525

(e) 1050

(f) None of the Above

31. At a certain burger joint, you can get 5 types of burger (veggie, turkey, buffalo, lean beef, juicy beef). Also, you can ask for any of six toppings on your hamburger. How many burger choices are possible?

(a) 144

(b) 320

(c) 336

$$5\left({_6}C_6 + {_6}C_5 + {_6}C_4 + {_6}C_3 + {_6}C_2 + {_6}C_1 + {_6}C_0\right)$$
$$= 5\left(1 + 6 + 15 + 20 + 15 + 6 + 1\right)$$
$$= 5(64)$$
$$= 320$$

(d) 525

(e) 1050

(f) None of the Above

32. Sheryl needs to staff a booth for the career fair. She has 8 employees who are willing to help at the booth. If she needs one employee to be at the booth for each of three time spots, in how many ways can she choose employees to staff the booth? Assume she will not use the same employee twice.

(a) 144

(b) 320

(c) 336

$$_8P_3 = 336$$

(d) 525

(e) 1050

(f) None of the Above

A.14 Be able to use counting principles (combinations, permutations, multiplication principle) and the rule for basic probabilities to determine probabilities. (2.3)

33. If you are dealt 4 cards from a standard deck, what is the probability that they are all the same suit?

(a) $\frac{13^4}{C(52,4)}$

(b) $\frac{4C(13,4)}{C(52,4)}$

(c) $\frac{C(13,4)}{P(52,4)}$

(d) $\frac{P(13,4)}{P(52,4)}$

(e) $\frac{13^4}{52^4}$

(f) None of the Above

$$\frac{{_{13}}C_4 + {_{13}}C_4 + {_{13}}C_4 + {_{13}}C_4}{{_{52}}C_4} = \frac{4 \times {_{13}}C_4}{{_{52}}C_4}$$

34. A (cheap) bike lock has 4 dials, each with six numbers. The 'code' for the lock consists of four numbers, and numbers can be repeated in the code. If someone trying to open the lock has time to try 24 codes, what is the probability they get the code and open the lock?

(a) $\frac{1}{18}$

$\frac{24}{6^4} = \frac{24}{1296} = \frac{1}{54}$

(d) $\frac{1}{36}$

(b) $\frac{1}{9}$

(e) $\frac{1}{54}$

(c) $\frac{1}{216}$

(f) None of the Above

35. A quiz consists of 3 multiple choice questions with 4 possible answers each. If a student guesses the answer to each question, what is the probability that they get all the answers correct?

(a) $\frac{1}{3^4}$

$\frac{1}{4^3}$

(d) $\frac{1}{C(4,3)}$

(b) $\frac{1}{4!}$

(e) $\frac{1}{2^4}$

(c) $\frac{1}{4^3}$

(f) None of the Above

36. An ice cream shop offers 20 flavors of ice cream. If 5 friends walk in and each get a single scoop of ice cream, what is the probability that at least two pick the same flavor?

(a) $1 - \frac{P(20,5)}{20^5}$

$1 - P(\text{same flavor})$

(d) $\frac{P(20,5)}{20^5}$

(b) $1 - \frac{5!}{P(20,5)}$

$1 - \left(\frac{20 P_5}{20^5}\right)$

(e) $\frac{1}{2}$

(c) $\frac{5!}{P(20,5)}$

(f) None of the Above

37. Ginny has 5 pairs of shoes (10 shoes total) in her closet. If she reaches in and grabs two shoes at random, what is the probability that she gets a matching pair?

(a) $\frac{1}{10}$

$\frac{5 C_1}{10 C_2} = \frac{5}{45} = \frac{1}{9}$

(d) $\frac{1}{2}$

(b) $\frac{1}{9}$

(e) $\frac{1}{3}$

(c) $\frac{1}{5}$

(f) None of the Above

38. A salesman has six prospects, including a person in Provo. If he randomly arranges his schedule to visit four of the six prospects, find the probability that the customer from Provo is NOT visited.

(a) $\frac{1}{10}$

(d) $\frac{1}{2}$

(b) $\frac{1}{9}$

(e) $\frac{1}{3}$

(c) $\frac{1}{5}$

(f) None of the Above

$$\frac{\# \text{ OF CITIES NOT VISITED}}{\text{TOTAL}} = \frac{2}{6} = \frac{1}{3}$$

01.e, 02.e, 03.b, 04.c, 05.e, 06.b, 07.a, 08.d, 09.d, 10.e, 11.d,
12.b, 13.e, 14.d, 15.e, 16.e, 17.d, 18.a, 19.e, 20.c, 21.a, 22.a,
23.b, 24.d, 25.a, 26.b, 27.a, 28.e, 29.e, 30.e, 31.b, 32.c, 33.b,
34.e, 35.c, 36.a, 37.b, 38.e, 39.d, 40.c, 41.b, 42.c