

# Answer Key for Exam A

Complete the following exam and record your answers on the bubble sheet provided.

1. What is the value of the  $x$  coordinate at the intersection of  $2x + 3y = 8$  and  $-x + 2y = 3$ ?

- (a) 4
- (b) 1
- (c) 2
- (d) 3
- (e) 0
- (f) None of the above.

ECHOLON:

$$\begin{array}{r}
 2x + 3y = 8 \\
 2(-x + 2y = 3) \\
 \hline
 -2x + 4y = 6 \\
 \hline
 7y = 14 \\
 y = 2
 \end{array}
 \qquad
 \begin{array}{r}
 2x + 3(2) = 8 \\
 2x + 6 = 8 \\
 2x = 2 \\
 x = 1
 \end{array}$$

2. The time an average resident of Some Town, CA spends commuting is normally distributed. A study found that the average commute time is 45 minutes a day. Also, 10% of commuters reported that their commute was 90 minutes or longer. What is the standard deviation for this distribution? (Round to the nearest minute.)

- (a) 25
- (b) 28
- (c) 30
- (d) 32
- (e) 35
- (f) None of the above.

$$z = \frac{x - \bar{x}}{\sigma}$$

$$\begin{array}{l}
 z = 1.28 \\
 \bar{x} = 45 \\
 x = 90 \\
 \sigma = ?
 \end{array}$$

$$\begin{array}{l}
 1.28 = \frac{90 - 45}{\sigma} \\
 1.28\sigma = 90 - 45 \\
 1.28\sigma = 45 \\
 \sigma = 35
 \end{array}$$

3. The Smith farm has 500 acres of land allotted for corn and wheat. The cost for cultivating corn and wheat (including seeds and labor) is \$42 and \$32 dollars an acre, respectively. Mr. Smith has \$18,000 available for cultivating these crops. If he wishes to use all the allotted land and his entire budget for cultivating these crops, how many acres of each should he plant?

- (a) 300 acres of corn, 200 acres of wheat.
- (b) 200 acres of corn, 300 acres of wheat.
- (c) 400 acres of corn, 100 acres of wheat.
- (d) 100 acres of corn, 400 acres of wheat.
- (e) 150 acres of corn, 350 acres of wheat.
- (f) None of the above.

ECHOLON:

$$\begin{array}{r}
 x + y = 500 \\
 42x + 32y = 18,000 \\
 -32(x + y = 500) \\
 \hline
 -32x - 32y = -16,000 \\
 \hline
 10x = 2,000 \\
 x = 200 \\
 200 + y = 500 \\
 y = 300
 \end{array}$$

4. Given the system of equations

$$x - 4y + 5z = 9$$

$$2y - 6z = 12$$

$$3y + z = 8$$

MULT. BY 6

Find  $z$ .

(a) 8

(b) 3

(c) 1.

(d) -1.

(e)  $10/8$ .

(f) None of the above.

$$\begin{array}{r} 2y - 6z = 12 \\ 18y + 6z = 48 \\ \hline 20y = 60 \end{array}$$

$$y = 3$$

$$2(3) - 6z = 12$$

$$6 - 6z = 12$$

$$-6z = 6$$

$$z = -1$$

5. Find the standard deviation for the sample 20, 22, 24, 26, 28.

(a)  $2\sqrt{2}$

(b)  $2\sqrt{3}$

(c)  $\sqrt{10}$

(d) 8

(e) 10

(f) None of the above.

$$\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$$

$$\bar{x} = 24$$

$$\sqrt{\frac{40}{4}} = \sqrt{10}$$

$$(20-24)^2 = 16$$

$$(22-24)^2 = 4$$

$$(24-24)^2 = 0$$

$$(26-24)^2 = 4$$

$$(28-24)^2 = 16$$

$$\frac{40}{4}$$

6. An unfair coin is flipped 20 times. On each toss the probability of getting heads is 0.7 whereas the probability of getting tails is 0.3. Let the random variable  $X$  be the number of heads minus the number of tails that occur in the 20 coin tosses. Find  $E(X)$ .

(a) 8

(b) 14

(c) 0.4

(d) 0

(e) 17

(f) None of the above.

$X$	20	20
$P(x)$	.7	.3
$E(x)$	14	6

$$14 - 6 = 8$$

7. Suppose that a basketball player makes free throws with a probability of 0.4. If she takes 4 shots, and each shot is independent, what is the expected number of points she will get? Assume that each free throw is worth one point.

(a) 1.0

(b) 2.0

(c) 1.6

(d) 2.4

(e) 3.0

(f) None of the above.

$$.4 + .4 + .4 + .4 = 1.6$$

8. An unfair coin with  $P(H) = 0.6$  is flipped 3 times and the result of each toss is noted. What is the probability that there are at least two heads?

- (a)  $(0.6)^3 + 3(0.6)^2(0.4)$   
 (b)  $(0.4)^3 + 3(0.4)^2(0.6)$   
 (c)  $3(0.6)^2(0.4)$   
 (d)  $(0.6)^2(0.4)$   
 (e)  $1 - 3(0.6)^3$   
 (f) None of the above.

3 HEADS:  $\binom{3}{3} (.6)^3 (.4)^0 = .6^3$   
 +  
 2 HEADS:  $\binom{3}{2} (.6)^2 (.4)^1 = 3(.6)^2(.4)$

9. The table below gives the values of the random variable  $X$  and the probability density function for  $X$ . Find the value of  $E(X)$ .

$x$	$P(X=x)$
-50	0.10
-10	$p_1$
0	0.20
10	0.35
50	0.20

$= .15$  ] MUST SUM TO 1

- (a) 10  
 (b) 7  
 (c) 5  
 (d) -10  
 (e) 12.5  
 (f) None of the above.

$(-50)(.10) = -5$   
 $(-10)(.15) = -1.5$   
 $(0)(.20) = 0$   
 $(10)(.35) = 3.5$   
 $(50)(.20) = 10$  + / 7

10. For which of the matrices below has the Gauss-Jordan method been completed?

$A = \begin{bmatrix} 1 & 0 & -1 & 3 & | & 1 \\ 0 & 1 & 4 & -1 & | & 1 \\ 0 & 0 & 0 & 1 & | & 3 \end{bmatrix}$        $B = \begin{bmatrix} 1 & 0 & -1 & 0 & | & 1 \\ 0 & 1 & 4 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 3 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 3 \end{bmatrix}$        $D = \begin{bmatrix} 1 & 0 & 2 & 0 & | & 1 \\ 0 & 1 & 1 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 3 \end{bmatrix}$

- (a)  $B$  only.  
 (b)  $B$  and  $D$  only.  
 (c)  $A$ ,  $B$  and  $D$  only.  
 (d)  $B$ ,  $C$  and  $D$  only.  
 (e)  $A$  and  $B$  only.  
 (f) None of the above.

TO BE "COMPLETED"  
 THE MATRIX MUST HAVE  
 AT LEAST  $X$  COLUMNS  
 COMPLETED IF THERE ARE  
 $X$  ROWS, IN THIS CASE, 3.

11. Which of the following graphs has a correlation coefficient  $r$  closest to 0?

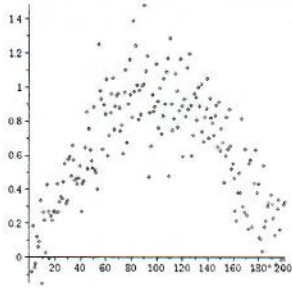


Figure A

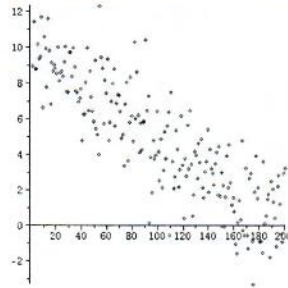


Figure B

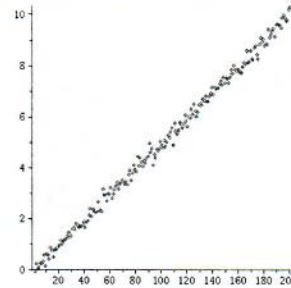


Figure C

- (a) A  
 (b) B  
 (c) C  
 (d) Both B and C have correlation coefficient near 0.  
 (e) Both A and B have correlation coefficient near 0.  
 (f) None of the above.

$r = 1 = \text{Perfect Line}$   
 $r = 0 = \text{No Line}$

12. Find the slope of the best fit line through the points:

$x$	$y$
1	1
3	2
4	3
5	4
7	5

- (a)  $\frac{7}{10}$   
 (b)  $\frac{5}{7}$   
 (c)  $\frac{7}{5}$   
 (d)  $\frac{10}{7}$   
 (e) 2  
 (f) None of the above.

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$\sum x = 20 \quad 1 + 3 + 4 + 5 + 7$$

$$n = 5$$

$$\sum xy = 74 \quad 1 + 6 + 12 + 20 + 35$$

$$\sum y = 15 \quad 1 + 2 + 3 + 4 + 5$$

$$\sum x^2 = 100 \quad 1 + 9 + 16 + 25 + 49$$

$$\sum (x)^2 = 400$$

$$\frac{5(74) - (20)(15)}{5(100) - 400} = \frac{70}{100} = \frac{7}{10}$$

13. Let  $C$  be a constant such that the equation  $4x + 6y = C$  describes a line passing through the point  $(3, 2)$ . Find the  $y$ -intercept of this line.

- (a)  $(0, -2/3)$   
 (b)  $(0, -3/2)$   
 (c)  $(0, -4)$   
 (d)  $(0, 4)$   
 (e)  $(0, -1/3)$   
 (f) None of the above.

$$\begin{aligned} 4(3) + 6(2) &= C \\ 12 + 12 &= C \\ 24 &= C \end{aligned}$$

$$\begin{aligned} 4(0) + 6y &= 24 \\ 6y &= 24 \\ y &= 4 \end{aligned}$$

14. A machine that fills orange juice cartons is set to fill them with 16.2 oz. If the actual contents of the cartons varies normally, with standard deviation 0.1 oz, and fills the cartons on average with 16.2 oz of juice, what percentage of the cartons contain less than 16 oz?

- (a) 2.28  
 (b) 3  
 (c) 4.18  
 (d) 5.30  
 (e) 6.81  
 (f) None of the above.

$$z = \frac{16 - 16.2}{.1}$$

$$z = -2$$

LOOK ON CHART AT  $-2 \rightarrow .0228$  OR  $2.28\%$

15. Three 200 pound people and one 100 pound person wait for an elevator. When it arrives, two of the four are selected at random to get on the elevator. What is the expected total weight of these two passengers?

- (a) 400  
 (b) 350  
 (c) 325  
 (d) 312.5  
 (e) 275  
 (f) None of the above.

	TWO 200 LBS.	ONE 100 LBS. ONE 200 LBS.
X	400	300
P(X)	$\frac{3C_2}{4C_2}$	$\frac{(1C_1)(3C_1)}{4C_2}$

$$\begin{aligned} &= 400(.5) + 300(.5) \\ &= 200 + 150 \\ &= 350 \end{aligned}$$

16. Each day Debbie has a 10% chance of finding a parking spot on campus. Find the probability that she will find a spot exactly 4 out of 5 days in a given week.

- (a)  $10(0.1)^1(0.9)^4$   
 (b)  $4(0.1)^1(0.9)^4$   
 (c)  $4(0.1)^4(0.9)^1$   
 (d)  $5(0.1)^4(0.9)^1$   
 (e)  $5(0.1)^1(0.9)^4$   
 (f) None of the above.

BINOMIAL:

$$\begin{aligned} &= \binom{n}{r} (p)^r (1-p)^{n-r} \\ &= \binom{5}{4} (.10)^4 (.90)^1 \\ &= \binom{5}{1} (.10)^4 (.90)^1 \end{aligned}$$

17. Find the median and mode for the following:

Value	Frequency
5	1
10	2
15	3
20	4
25	5
30	6

- (a) median: 20, mode: 30  
 (b) median: 20, mode: 30  
 (c) median: 25, mode: 30  
 (d) median: 25, mode: 25  
 (e) median: 30, mode: 25  
 (f) None of the above.

18. A box contains 2 nickels and 3 quarters. Two coins are selected simultaneously and at random, and a random variable  $X$  is defined as the total value (in cents) of the two coins selected. Find the probability  $P(X = 30)$ .

- (a)  $1/2$   
 (b)  $3/10$   
 (c)  $11/20$   
 (d)  $9/20$   
 (e)  $3/5$   
 (f) None of the above.

$$\frac{6}{5} = \frac{{}^2C_1 {}^3C_1}{{}^5C_2} = \frac{6}{10} = \frac{3}{5}$$

19. Fill in the missing values  $a, b, c$  in the probability density chart below:

$x$	$P(X = x)$	$x \cdot P(X = x)$
$a$	$b$	$1/2$
1	$c$	$1/3$
-2	$1/3$	$-2/3$

What is the value of  $a$ ?

- (a) 3  
 (b)  $2/3$   
 (c) 2  
 (d)  $3/2$   
 (e) 6  
 (f) None of the above.

1. SOLVE FOR C :  $1 \times c = 1/3$   $c = 1/3$   
 2. SOLVE FOR B :  $b + c + 1/3 = 1$   $b = 1/3$   
 3. SOLVE FOR A :  $a \times b = 1/2$   $a = 3/2$

20. A certain course has three midterm exams. A student scores 80 on their first exam and 92 on their second exam. What score must they get on their third exam if they want the mean of their exam scores to be 90?

- (a) 100  
 (b) 99  
 (c) 98  
 (d) 97  
 (e) 96  
 (f) None of the above.

$$\frac{80 + 92 + x}{3} = 90$$

$$172 + x = 270$$

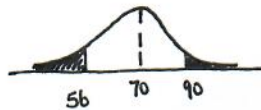
$$x = 98$$

21. The augmented matrix shown has been obtained from a system of equations. Decide which of the following statements is true about the associated system of equations.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -3 & 2 \\ 0 & 2 & -6 & 4 \end{array} \right] \xrightarrow{-\frac{1}{2}R_3 + R_2} R_3 \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- (a) The system has a unique solution.  
 (b) The system has no solution.  
 (c) The system has an infinite number of solutions with one arbitrary parameter.  
 (d) The system has an infinite number of solutions with two arbitrary parameters.  
 (e) The system has an infinite number of solutions with three arbitrary parameters.  
 (f) None of the above.
22. On a certain freeway, the mean speed is 70 mph with standard deviation 8 mph. What percentage of drivers drive slower than 56 mph or faster than 90 mph?

- (a) 3.94  
 (b) 4.29  
 (c) 3.19  
 (d) 5.12  
 (e) 4.63



- (f) None of the above.

$$z = \frac{56 - 70}{8}$$

$$z = \frac{90 - 70}{8}$$

$$z = -1.75$$

$$z = 2.5$$

$$\% = .0401$$

$$\% = .9938$$

$$.0401 + .0062 = 4.63\%$$

23. A theater charges \$8 for main floor seats and \$5 for balcony seats. If all seats are sold, the ticket income is \$4200. At one show, 25% of the main floor seats and 40% of the balcony seats were sold at ticket income was \$1200. How many seats are on the main floor, and how many are on the balcony?

- (a) 500 main, 100 balcony  
 (b) 300 main, 300 balcony  
 (c) 100 main, 500 balcony  
 (d) 200 main, 400 balcony  
 (e) 400 main, 200 balcony  
 (f) None of the above.

$$1) 8x + 5y = 4200$$

$$2) (.25)8x + (.40)5y = 1200$$

$$\rightarrow 2x + 2y = 1200$$

$$\rightarrow x + y = 600$$

$$x + 200 = 600$$

$$x = 400$$

$$8x + 5y = 4200$$

$$-8x - 5y = -4200$$

$$0 - 3y = -600$$

$$y = 200$$

24. Find the  $x$ -intercept of the line which contains the point  $(3, 1)$  and which is parallel to the line  $2x - 3y = 5$ .

- (a)  $(-1, 0)$   
 (b)  $(1, 0)$   
 (c)  $(0, 0)$   
 (d)  $(-3/2, 0)$   
 (e)  $(3/2, 0)$   
 (f) None of the above.

$$2x - 3y = 5$$

$$-3y = -2x + 5$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

$$m = \frac{2}{3}$$

$$1 = \left(\frac{2}{3}\right)(3) + b$$

$$1 = 2 + b$$

$$b = -1$$

$$y = \frac{2}{3}x - 1$$

$$0 = \frac{2}{3}x - 1$$

$$1 = \frac{2}{3}x$$

$$x = \frac{3}{2}$$

25. Find the area under the standard normal curve between  $z = -0.23$  and  $z = 2.34$ .

- (a) .5378  
 (b) .5579  
 (c) .5748  
 (d) .5814  
 (e) .6253  
 (f) None of the above.



$$\text{---} .9904 \text{---}$$

$$\text{---} .4090 \text{---}$$

$$\begin{array}{r} .9904 \\ - .4090 \\ \hline .5814 \end{array}$$

### EXPECTED VALUE:

$$E(x) = x_1 p_1 + x_2 p_2 + \dots$$

### BINOMIAL PROBABILITY

$${}^n C_x \cdot p^x \cdot (1-p)^{n-x}$$

### RANDOM VARIABLE

- 1.) FIND PROBABILITY
- 2.) FIND EXPECTED VALUE

### STANDARD DEVIATION

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$z = \frac{x - \bar{x}}{\sigma}$$

### LEAST SQUARE

$$m = \frac{n \sum xy - \sum x \sum y}{n \sum (x^2) - \sum x^2}$$

$$b = \frac{\sum y - m \sum x}{n}$$

$$cc = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum (x^2) - \sum x^2} \times \sqrt{n \sum (y^2) - \sum y^2}}$$