Math 118: Exam 3

RED

Name: Samuel S.

Complete the following exam and record your answers on the bubble sheet provided.

1. Find the maximum value of 2x + y on the feasible set given by the constraints $x + 10y \le 50$, $x + y \le 5$, $10x + y \le 50$, $x \ge 0$, $y \ge 0$.



y < -1/0x +5 y < -x +5 y < -10x +50

$$(0,5) = 2(0)+5(1)+5$$

 $(5,0) = 2(5)+1(0)=10$



2. Suppose that x and y are numbers such that

MILT. MATRICIES
$$\begin{array}{ll} \text{I:} \left(\mathbb{R}, \times \mathbb{C},\right) + \left(\mathbb{R}, \times \mathbb{C},\right) \\ \text{2. Set sum} &= \begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix} \end{array}$$

Find x:

(a)
$$\frac{2}{(b)}$$
 (b) -4 $0 + 6 + 3$

$$\begin{bmatrix} -11 + 3x + 2x & -y + x + 0 \\ 0 + 6 + 3x & 0 + 2 + 0 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 18 & 2 \end{bmatrix}$$

(c) 6
$$-11 + 5x = 9$$
 (d) 4

- (f) None of the above.
- 3. A linear programming problem in standard maximum form has the initial tableau

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

(a)
$$(0,1,2)$$

(c)
$$(12, 20, 60)$$

- 4. Kathy needed to find the inverse to a 3 by 3 matrix A, so she started putting the augmented matrix [A|I] into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:
 - I. FINISH SO $\begin{bmatrix} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$ 18

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- -2(a)
- (b)
- (c)
- (d) 1
- (e) 6
- (f) None of the above.
- 5. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.
 - s [.28 .4 4 .3 .29]

 - (e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
 - None of the above.
- 6. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and soal-must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
 - 700 steel, 900 lumber (a)
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - None of the above. (f)

1. TOP TO BOTTOM

1. ECONOMIC/LEONTIEF

2.
$$(\mathbf{I} - \mathbf{A})^{-1} \mathbf{D} = \mathbf{X}$$
 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 16 \\ -5 & 16 \end{bmatrix} = \begin{bmatrix} .8 & -.6 \\ -.5 & .4 \end{bmatrix}$

 $R_1 - 2R_3$ | 0 0 | 4 6 | 0 | 0 - 1 4 - 2 | R₂ + R₃ | 0 0 | 0 - 1 | 1

3. SOLVE [.8 .6] 1 0] .5R, +.8R2 > [.8 -.6 | 1 0] 30R2 + R,

$$\rightarrow \begin{bmatrix} .8 & 0 & | & 16 & 24 \\ 0 & .02 & | & .5 & .8 \end{bmatrix} / .8 \rightarrow \begin{bmatrix} 20 & 30 \\ 25 & 40 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 30 \\ 25 & 40 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 700 \\ 900 \end{bmatrix}$$

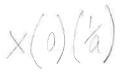
7. If
$$A$$
 is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$



find the (1,2) entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix},$$



(a)
$$-2$$

$$1. (XA) \cdot A^{-1} = X$$

(e)

2. A' GOES AFTER, BECAUSE A

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 5 & -4 \\ 1 & -11 & -5 \\ 1 & 3 & -4 \end{bmatrix}$$

(f) None of the above.

8. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \ge 0, y_2 \ge 0, 2y_1 + 3y_2 \ge 6$, and $2y_1 + y_2 \ge 7$. What is the sum $y_1 + y_2$?

| | | | , | 0 - | · | | | | | 91 | 94 |
|-----|---|--|---|---|---|----|----|----|---|----|-------|
| (a) | 7 | 1. ORIGINAL | 2 | 3 | 6 | | 2 | 2 | ١ | 0 | 0 |
| (b) | 3 | 2. TRANSVERSE | 2 | 1 | 7 | Ms | 3 | 1 | 0 | 4 | 0 |
| (0) | J | 3. NEGATIVES | 5 | 2 | 0 | | -6 | -7 | 0 | 0 | 1 |
| / \ | ~ | A CONTRACTOR OF THE CONTRACTOR | - | 100000000000000000000000000000000000000 | | | _ | | | | 1,477 |

(c)

(f) None of the above.

9. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \ge 0$, $y_2 \ge 0$, $y_3 \ge 0$, $y_1 + y_2 + y_3 \ge 100$, and $2y_1 + y_2 \ge 50$. Solve the dual problem and report its solution variables x_1 and x_2 .

(a)
$$x_1 = 2, x_2 = 0$$

(b)
$$x_1 = 1, x_2 = 0$$

(c)
$$x_1 = 0, x_2 = 1$$

(d)
$$x_1 = 0, x_2 = 0$$

(e)
$$x_1 = 1, x_2 = 1$$

Y3 = 0

10. Given

$$6\begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2\begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

 $\begin{bmatrix} -6 & 18 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2x \\ 10 & -8 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 22 & -8 \end{bmatrix}$

Find x.

- (b) 10
- (c) 1
- (d) -1
- (e) 8
- (f) None of the above.

11. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

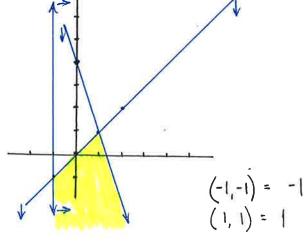
be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: X = AX + D]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- $X = AX + D] \qquad (I A)^{-1} = X$ $\begin{bmatrix} 1 & O \\ O & 1 \end{bmatrix} \begin{bmatrix} .4 & .2 \\ .8 & .2 \end{bmatrix} = \begin{bmatrix} .6 & .2 \\ -.8 & .8 \end{bmatrix}$ $\begin{bmatrix} .6 & -.2 \\ -.8 & .8 \end{bmatrix}^{-1} = \begin{bmatrix} 2.5 & .625 \\ 2.5 & 1.875 \end{bmatrix}$
- $\begin{bmatrix} 2.5 & .625 \\ 2.5 & 1.875 \end{bmatrix} \begin{bmatrix} 20 \\ 80 \end{bmatrix} = \begin{array}{c} \times_{1} & = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$
- (f) None of the above.

12. Find the minimum value of the function 3x-2y on the set $x \ge -1$, $x-y \ge 0$, $3x+y-4 \le 0$.

- (a) 0
- (b) -17
- (c) -5
- (d) -1
- (e) -11
- (f) None of the above.





13. Find the (1,2) entry in the matrix 2A - B where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a)
- (b)
- (c) 5
- (d) 6
- 7 (e)
- (f) None of the above.
- 14. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
 - BC(a)
 - (b) A+C
 - CA(c)
 - (d) AB
 - (e) BB
 - (f) None of the above.



 $\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \end{bmatrix} \qquad \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \end{bmatrix}$

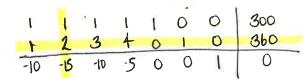
 $\begin{bmatrix} -4 & 6 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 3 & 2 \end{bmatrix}$

- MULT. THE SAME MIDDLE
- 2. ADDING: DENTICAL BE
- 15. Find the minimum of x-2y on the feasible set given by the constraints: $x \ge 0, y \ge 0$, $x + y \ge 3, y \le 3, x - y \le 3$.
 - (a)

- (c)
- (d)
- There is no solution to this problem. (e)
- (f) None of the above.
- 16. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$, $x_4 \ge 0$, $x_1 + x_2 + x_3 + x_4 \le 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \le 360$.
 - (a) 6600
 - (b) 2400
 - (c) 5400
 - (d) 900
 - (e) 3300
 - (f) None of the above.



10



- 2R, R2 10-1-22-10-240 15R2+2R3 -50 25 50 0 15 2 5400

50.60= 3000

17. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.

(a) Maximize 12x + 18y, subject to $x + 2y \le 100$, $40x + 90y \le 50$, $x \ge 0$, $y \ge 0$

(b) Maximize 18x + 12y, subject to $x + 2y \le 100$, $40x + 90y \le 3000$, $x \ge 0$, $y \ge 0$

(e) Maximize 12x + 18y, subject to $2x + 2y \le 100$, $40x + 90y \le 3000$, $x \ge 0$, $y \ge 0$

(d) Maximize 12x + 18y, subject to $2x + 2y \le 100$, $90x + 40y \le 50$, $x \ge 0$, $y \ge 0$

(e) Maximize 12x + 18y, subject to $x + 2y \le 100$, $40x + 90y \le 3000$, $x \ge 0$, $y \ge 0$

(f) None of the above.

18. Consider the problem: "A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y, respectively, should be planted to maximize revenue?" Which of the following is not a constraint for this problem:

e) $x + 2y \le 3000$ d) $x + y \le 2000$ $90x + 60y \le 150,000$

 $60x + 90y \le 150,000$ 170x + 190y

(f) None of the above.

- 19. Consider the problem: "A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met." Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
 - (a) 30x + 25y
 - (b) x + 0.5y 2A + 5B
 - (c) 2x + 5y
 - (d) 2x + 25y
 - (e) 30x + 0.5y
 - (f) None of the above.

- 20. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \ge 0$, $x_2 \ge 0$, $2x_1 + 4x_2 \le 15$, and $3x_1 + x_2 \le 10$. What is $x_1 - x_2$?
 - (a)
 - 5/2(b)
 - (c) 5
 - (d) 35/2
 - (e) 2
 - None of the above. (f)
- 2 4 1 0 0 15 3R1-2R2 0 10 3 -2 0 25 3 1 0 1 0 10 3 10 10 10 -5 -2 0 0 1 0 5R2-2R3 0 1 0 5 3 50
- 0 10 3 -2 0 25 30x, = 75 X, = 2.5 10R₂-R₁ 30 0 -3 12 0 75 10x₂ = 2.5 X₂ = 2.5
- 21. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

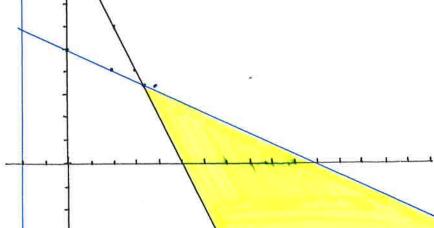
$$\begin{bmatrix} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{bmatrix}$$

After the next pivot, how much does the objective function increase?

- 64 (a)
- (b) 76
- (c) 12
- (d)
- (e) The objective does not increase.
- (f) None of the above.
- 22. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \ge -2$, $y \ge -3$, $x + 2y \le 10$, $2x + y \ge 10$.
 - (a)
 - (b) 1
 - (c)
 - (d) 3
 - (e)
 - None of the above. (f)
- Y = -2x +5 Y = -2x + 10

5R,-3R2 15 2R,+R3 7h





23. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\begin{bmatrix} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ -2 & 5 & -6 & 1 & 64 \end{bmatrix}$$

Which element of the tableau should you pivot on next?

- (a) (2,1)
- (b) (2,3)
- (c) (3,1)
- (d) (3,2)
- (e) Pivoting is not the correct action for this tableau.
- (f) None of the above.
- 24. What is the (2,1) entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.
 - (a) 1
 - (b) -1
 - (c) -2
 - (d) -3
 - (e) 3
 - (f) None of the above.
- $\begin{bmatrix} 1 & 1 & | & 1 & 0 \\ 3 & 2 & | & 0 & 1 \end{bmatrix} & 3R_1 R_2 \rightarrow \begin{bmatrix} 1 & | & 1 & 0 \\ 0 & | & 3 & -1 \end{bmatrix}$ $R_2 R_1 \begin{bmatrix} -1 & 0 & | & 2 & -1 \\ 0 & | & 3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & -2 & 1 \\ 0 & | & 3 & -1 \end{bmatrix}$
- 25. A feasible set for a linear programming problem is defined by $x \ge 0$, $y \ge 0$, $2x + y \le 12$, $4x + 4y \le 44$. Which of the following is *not* a corner point of the feasible set?
 - (a) (1,10)
 - (b) (0, 12)
 - (c) (0,0)
 - (d) (0,11)
 - (e) (6,0)
 - (f) None of the above.