FINAL

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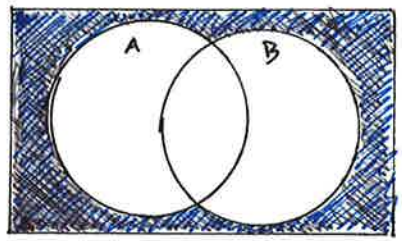
Answer Key for Exam $\boxed{A}$

ATTENTION: This prior exam is only a sample of **SOME** problems we could ask to test your knowledge of **SOME** concepts covered in this class. Please see the Final Exam Review for a **COMPLETE** list of concepts that can be tested on the final exam. Also, see the Final Exam Review for Exam policies and make sure you check the testing center webpage for Final Exam locations.

1. For two sets A and B, which of the following is always true about the set $(A \cup B)'$?

   (a) $(A \cup B)' = A' \cup B'$
   (b) $(A \cup B)' = A' \cup B$
   (c) $(A \cup B)' = A \cup B$
   (d) $(A \cup B)' = A \cap B$
   (e) $(A \cup B)' = A' \cap B'$
   (f) None of the Above



2. A bridge hand is made up of 13 cards from a standard deck of 52 cards. What is the probability that a hand chosen at random has at least 3 aces?

   (a) $\frac{C(4,3)C(48,10)+C(48,9)}{C(52,13)}$
   (b) $\frac{4^3 \cdot 48^{10}}{C(52,13)}$
   (c) $\frac{4!48^{10}}{52^{13}}$
   (d) $\frac{4^3 \cdot 48^{10}}{52^{13}}$
   (e) $\frac{C(4,3)C(48,10)}{52^{13}}$
   (f) None of the Above

   C(#Aces, #Chosen) C(#not Aces, #not chosen)

   1. $\frac{E}{S}$

   2. Order does not matter

   $= \frac{C(4,3)C(48,10) + C(4,4)C(48,9)}{C(52,13)}$

3. Janie works for a restaurant which has 7 entrees, of which 3 are beef, 2 are chicken, and 2 are vegetarian. She is putting together a corporate catering menu and needs to choose 3 entrees to put on the menu. If she chooses the menu randomly, what is the probability that she has one of each type of entree?

   (a) $\frac{1}{35}$
   (b) $\frac{1}{210}$
   (c) $\frac{1}{420}$
   (d) $\frac{8}{35}$
   (e) $\frac{12}{35}$
   (f) None of the Above

   3 Beef
   2 Chicken
   2 Veg.
   7

   $\frac{E}{S} = \frac{{}_3C_1 \cdot {}_2C_1 \cdot {}_2C_1}{{}_7C_3} = \frac{3 \cdot 2 \cdot 2}{35} = \frac{12}{35}$

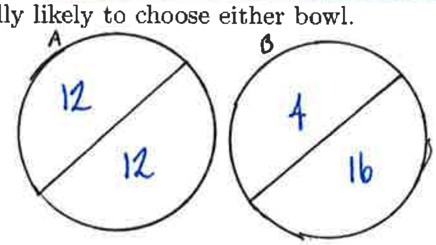
4. At Skinny Jill's Pizza, there are three crust options and 6 available toppings. You may order as many toppings as you like, but they do not allow you to order double toppings. How many different pizzas can be ordered?

(a) 90
(b) 96
(c) 108
(d) 192
(e) 216
(f) None of the Above

$$_6C_6 = 1 + {}_6C_5 = 6 + {}_6C_4 = 15 + {}_6C_3 = 20 + {}_6C_2 = 15 + {}_6C_1 = 6 + {}_6C_0 = 1 = 64 \times \underset{\text{CRUSTS}}{3} = 192$$

5. Grandma has put out two bowls of candy for the holidays. The first bowl has 12 mint truffles and 12 raspberry truffles. The second bowl has 4 mint truffles and 16 peanut butter truffles. I picked a bowl at random, then picked a truffle at random, and got a mint truffle. What is the probability that I picked the first bowl, given that I got a mint truffle? You may assume that I was equally likely to choose either bowl.

(a) 5/6
(b) 5/7
(c) 5/8
(d) 5/9
(e) 1/2
(f) None of the Above



$$p(A|\text{Mint}) = \frac{p(A)\cdot p(M|A)}{p(A)p(M|A) + p(B)p(M|B)} = \frac{.5 \times .5}{(.5)(.5) + (.5)(.2)} = \frac{.25}{.35} = \frac{5}{7}$$

6. If A and B are independent events with $P(A) = 0.2$, $P(B) = 0.5$, find $P(A \cup B)$.

(a) 0.1
(b) 0.25
(c) 0.6
(d) 0.7
(e) 0.75
(f) None of the Above

Union Rule: $p(E \cup F) = p(E) + p(F) - p(F \cap E)$

$= .2 + .5 - (A \times B)$

$= .2 + .5 - .1 = .6$

7. Two dice are rolled and the sum is 8. What is the probability that one of the dice is a 6?

(a) 2/5
(b) 6/36
(c) 1/2
(d) 2/3
(e) 1/6
(f) None of the Above

$\frac{E}{S}$

2, 6
3, 5
4, 4
5, 3
6, 2

$2/5$

$$\frac{\text{possible successes}}{\text{possible outcomes}}$$

8. The letters 'b,a,n,a,n,a' are written on index cards which are then shuffled thoroughly. What is the probability that, when the cards are dealt out, they spell the word 'banana'.

(a) $\frac{1}{60}$

(b) $\frac{1}{360}$

(c) $\frac{1}{720}$

(d) $\frac{1}{30}$

(e) $\frac{1}{90}$

(f) None of the Above

B : 1
A : 3
N : 2

$$\frac{\#\text{ successes}}{\frac{\#\text{ letters!}}{\text{repeated!}}} = \frac{1}{\frac{6!}{3!2!}} = \frac{1}{\frac{720}{12}} = \frac{1}{60}$$

9. The table below gives the values of the random variable X and the density function for X. Find the value of $E(X)$

x	$P(X = x)$
-5	0.1
-1	0.2
0	0.2
1	p_1 = .3
5	0.2

-.5
-.2
0
.3
1
.6

(a) 0

(b) 0.3

(c) 0.5

(d) 0.6

(e) 0.7

(f) None of the above.

* Probability must sum to 1

10. Find the standard deviation for the sample 4, 2, 6, 9, 4.

(a) $\sqrt{7}$

(b) $\sqrt{\frac{28}{5}}$

(c) $\frac{2\sqrt{7}}{5}$

(d) $\frac{3\sqrt{17}}{5}$

(e) $\frac{3\sqrt{17}}{4}$

(f) None of the above

$\bar{X} = 5$

$$\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(4-5)^2 + (2-5)^2 + (6-5)^2 + (9-5)^2 + (4-5)^2}{4}}$$

$$= \sqrt{\frac{1 + 9 + 1 + 16 + 1}{4}}$$

$$= \sqrt{\frac{28}{4}}$$

$$= \sqrt{7}$$

11. Suppose that a football player makes field goals with a probability of 0.9. If she has 4 attempts in a game, and each attempt is independent, what is the probability that she makes at least 2 field goals? (Choose the closest answer)

(a) 0.9963
(b) 0.6561
(c) 0.9477
(d) 0.9999
(e) 0.6562
(f) None of the above.

Binomial: $C(n,x)\cdot(p)^x\cdot(1-p)^{n-x}$

$C(4,2)\cdot(.9)^2\cdot(.1)^2 = .0486$
$+\ C(4,3)\cdot(.9)^3\cdot(.1)^1 = .2916$ $= .9963$
$+\ C(4,4)\cdot(.9)^4\cdot(.1)^0 = .6561$

12. For which of the matrices below has the Gauss-Jordan method been completed?

$$A = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad B = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 1 \\ 0 & 1 & 4 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$C = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad D = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

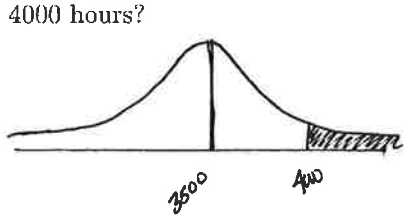
(a) A only
(b) A and C only
(c) A, B and C only
(d) A, C and D only.
(e) A and B only
(f) None of the above.

"Completed" = min # of completed rows

13. A light bulb manufacturer has a line of light bulbs that last an average of 3500 hours, with a standard deviation of 400 hours. Assuming a normal distribution, what is the probability that the light bulb will last at least 4000 hours?

(a) 0.0668
(b) 0.1056
(c) 0.1245
(d) 0.1587
(e) 0.2112
(f) None of the above

$\sigma = 400$



$Z = \dfrac{X - \bar{X}}{\sigma}$

$1.0000 - .8944 = .1056$

$Z = \dfrac{4000 - 3500}{400} = 1.25 \longrightarrow .8944$

14. A chuck wagon charges \$8 for a combo meal and \$5 for a salad. If they sell their entire inventory by the end of the day, they will take in \$3700 in revenue. If they only take in \$800 in revenue by selling 25% of their combo meals and 20% of their salads, how many salads did they have in inventory?

(a) 100
(b) 150
(c) 200
(d) 300
(e) 500
(f) None of the above.

$X = \text{Combo}$
$Y = \text{Salad}$

$8x + 5y = 3700$
$8(.25)x + 5(.2)y = 800$
$\rightarrow 2x + y = 800$

$-8x - 4y = -3200$
$8x + 5y = 3700$
$y = 500$

15. A box contains 3 dimes and 2 quarters. Two coins are selected simultaneously and at random, and a random variable X is defined as the total value (in cents) of the two coins selected. Find $E(X)$.

(a) 35
(b) 32
(c) 30
(d) 26
(e) 40
(f) None of the above.

X	20	35	50	
P(x)	$\frac{C(3,2)C(2,0)}{C(5,2)}$ .3	$\frac{C(3,1)C(2,1)}{C(5,2)}$ .6	$\frac{C(3,0)C(2,2)}{C(5,2)}$ .1	
=	6	21	5	= 32

16. Given the system of equations

$$x - 4y + 5z = 9$$
$$2y - 6z = 12$$
$$3y + z = 8$$

$y = 3z + 6$

Find x.

(a) 8
(b) 26
(c) 3.
(d) -1.
(e) 3/8.
(f) None of the above.

$3(3z + 6) + z = 8$
$9z + 18 + z = 8$
$10z = -10$
$z = -1$

$3y - 1 = 8$
$3y = 9$
$y = 3$

$x - 4(3) + 5(-1) = 9$
$x - 12 - 5 = 9$
$x - 17 = 9$
$x = 26$

17. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$AXA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

$A^{-1} \cdot AXA = XA$

$XA \cdot A^{-1} = X$

$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -2 & 2 \\ 9 & 11 & -9 \\ 4 & 5 & -4 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -2 & 2 \\ 9 & 11 & -9 \\ 4 & 5 & -4 \end{pmatrix}\begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -4 & 2 \\ 2 & 28 & -17 \\ 1 & 13 & -8 \end{pmatrix}$$

(a) -4

(b) 1

(c) 5

(d) -2

(e) -1

(f) None of the above.

18. Find the values y_1, y_2 and y_3 that minimize the objective $y_1 + y_2 + y_3$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $3y_1 + 4y_3 \geq 16$, and $4y_1 + 3y_2 + 2y_3 \geq 12$. What is y_1?

$$\begin{array}{ccc|c} 4 & 3 & 2 & 12 \\ 3 & 0 & 4 & 16 \\ \hline 1 & 1 & 1 & 0 \end{array} \rightsquigarrow \begin{array}{cccccc|c} 3 & 4 & 1 & 0 & 0 & 0 & 1 \\ 0 & 3 & 0 & 1 & 0 & 0 & 1 \\ 4 & 2 & 0 & 0 & 1 & 0 & 1 \\ \hline -16 & -12 & 0 & 0 & 0 & 1 & 0 \end{array}$$

$= \begin{array}{cccccc|c} 0 & 0 & 8 & 0 & 14 & 5 & 22 \end{array} /5 = 8/5 = y_1$

(a) 8/5

(b) 0

(c) 8/3

(d) 5

(e) 3/8

(f) None of the above.

19. Find the variable x satisfying the equation below

$$\begin{pmatrix} -1 & 2 & x \\ 5 & 2 & 7 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 0 \\ x & 6 \end{pmatrix} = \begin{pmatrix} 13 & -26 \\ -25 & 52 \end{pmatrix}$$

$-1 - 2 + x^2 = 13$

$x^2 = 16$

$x = \pm 4$

$5 - 2 + 7x = -25$

$7x = -28$

$x = -4$

(a) 2

(b) -4

(c) 6

(d) 4

(e) -5

(f) None of the above.

20. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x + 4y \geq 12$, $x - y \leq 0$, $2y - x \leq 6$, $x \leq 6$.

(a) 1

(b) 2

$\boxed{\text{(c)}}$ 3

(d) 4

(e) 5

(f) None of the above.

$y \geq -\frac{1}{4}x + 3$ $y \geq x$ $y \leq \frac{1}{2}x + 3$

* Graph

21. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 4 & 0 & 6 \\ 0 & 3 & 1 & 3 & 3 & 0 & 15 \\ 0 & 5 & 0 & 1 & -12 & 1 & 70 \end{array}\right]$$

After the next pivot, how much does the objective function increase?

(a) 82

(b) 60

$\boxed{\text{(c)}}$ 18

(d) 12

(e) The objective does not increase.

(f) None of the above.

$3R_1 + R_3$

$18 + 70 = 88$

22. Find the maximum of $8x_1 + 3x_2 + x_3$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $7x_1 + 6x_2 + 8x_3 \leq 118$, and $4x_1 + 5x_2 + 10x_3 \leq 220$.

(a) 102

(b) 118/7

(c) 826

$\boxed{\text{(d)}}$ 944/7

(e) 1660

(f) None of the above.

$$\left[\begin{array}{cccccc|c} 7 & 6 & 8 & 1 & 0 & 0 & 118 \\ 4 & 5 & 10 & 0 & 1 & 0 & 220 \\ \hline -8 & -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$7R_2 - 4R_1$

$8R_1 + 7R_3$

$$\left[\begin{array}{cccccc|c} 7 & 6 & 8 & 1 & 0 & 0 & 118 \\ 0 & 11 & 38 & -4 & 7 & 0 & 1068 \\ \hline 0 & 27 & 57 & 8 & 0 & 7 & 944 \end{array}\right]$$

$= 944/7$

23. Which of the following is not a constraint for this problem: "Certain animals in a rescue shelter must have at least 30g of protein at at least 20g of fat per feeding period. These nutrients come from food A, which costs 18 cents per unit and supplies 2g of protein and 4g of fat; and food B, which costs 12 cents per unit and has 6g of protein and 2g of fat. Food B is bought under a long-term contract requiring that at least 2 units of B be used per serving. Another contract requires that the amount of food B used be no more that 3 times the amount of food A used. How much of each food must be bought to produce the minimum cost per serving?" Let x be the number of units of food A and y be the number of units of food B.

(a) $2x + 6y \geq 30$

(b) $4x + 2y \geq 20$

$\boxed{\text{(c)}}$ $2x + 4y \leq 30$

(d) $2 \leq y$

(e) $y \leq 3x$

(f) None of the above.

$2x + 6y \geq 30$

$4x + 2y \geq 20$

24. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of lumber to produce one unit of lumber. How many units of lumber must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?

(a) 1000 lumber

$\boxed{\text{(b)}}$ 900 lumber

(c) 700 lumber

(d) 200 lumber

(e) 180 lumber

(f) None of the above.

$(I - A)^{-1} D = X$

$A = \begin{bmatrix} .2 & .6 \\ .5 & .6 \end{bmatrix}$ $D = \begin{bmatrix} 20 \\ 10 \end{bmatrix}$

$= \begin{bmatrix} .8 & -.6 \\ -.5 & .4 \end{bmatrix}$ inverse $\rightarrow$ $\begin{bmatrix} 20 & 30 \\ 25 & 40 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 700 \\ 900 \end{bmatrix}$ $\begin{array}{l} \text{steel} \\ \text{lumber} \end{array}$

25. Determine whether the matrix $\begin{bmatrix} 1/3 & 0 & 2/3 \\ 1/4 & 3/4 & 0 \\ 2/5 & 2/5 & 1/5 \end{bmatrix}$ is a regular matrix.

(a) This matrix is regular because there is no absorbent state.

(b) This matrix is regular because for the vector $V = \begin{bmatrix} 6/19 & 8/19 & 5/19 \end{bmatrix}$, $VA = V$.

(c) This matrix is not regular because it is not a transition matrix.

$\boxed{\text{(d)}}$ This matrix is regular because A^2 has only positive entries.

(e) This matrix is not regular because A^2 has a 0 entry in the $(1, 2)$ location.

(f) None of the Above

Regular:
- Rows add up to 1
- All positive values

$A^2 A = A^3$

26. A community has a goal of reducing their carbon footprint. They have offered incentives to homeowners to convert to solar heating. Community leaders can convince 20% of those who use fossil fuel based energy to convert to solar each year. Of those who use solar power, 10% convert back to fossil fuels annually. Originally, 90% of homes are heated by fossil fuels and 10% use solar heating. What is the percentage of fossil fuel users after 2 years?

(a) 0.389
(b) 0.591
(c) 0.611
(d) 0.683
(e) 0.73
(f) None of the Above

$$\begin{array}{c c} & \begin{array}{cc} F & S \end{array} \\ \begin{array}{c} F \\ S \end{array} & \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix}^2 \end{array} = \begin{bmatrix} .66 & .34 \\ .17 & .83 \end{bmatrix} \begin{bmatrix} .9 & .1 \end{bmatrix} = \begin{bmatrix} .611 & .389 \end{bmatrix}$$

27. Each term, the instructor of a course has to decide whether the final exam should be timed or not. If it was timed last term, there is a 90% chance that it will be timed this term. If it was not timed last term, there is a 90% chance that it will not be timed this term. Find the long range average as to what percentage of the exams are timed.

(a) 90%
(b) 81%
(c) 75%
(d) 50%
(e) This Markov chain does not have an equilibrium vector.
(f) None of the Above.

$$\begin{array}{c c} & \begin{array}{cc} T & NT \end{array} \\ \begin{array}{c} T \\ NT \end{array} & \begin{bmatrix} .9 & .1 \\ .1 & .9 \end{bmatrix} \end{array}$$

$$.9x + .1y = x$$
$$.1x + .9y = y$$

$$-.1x + .1y = 0$$
$$\cancel{.1x - .1y = 0}$$
$$x + y = 1$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & 1 & 1 \end{array}\right]$$

$$\underset{R_1+R_2}{=} \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 2 & 1 \end{array}\right]$$

$$\underset{R_2-2R_1}{=} \left[\begin{array}{cc|c} 2 & 0 & 1 \\ 0 & 2 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

28. Suppose you are solving for the equilibrium vector of a Markov chain with transition matrix P given below. Which system of equations below could you solve to find the equilibrium vector?

$$\begin{bmatrix} \frac{7}{10} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{10} & \frac{3}{10} & \frac{3}{5} \\ \frac{1}{5} & \frac{1}{2} & \frac{3}{10} \end{bmatrix}$$

(a) $\begin{cases} -\frac{3}{10}x_1 + \frac{1}{10}x_2 + \frac{1}{5}x_3 = 0 \\ \frac{1}{5}x_1 - \frac{7}{10}x_2 + \frac{1}{2}x_3 = 0 \\ \frac{1}{10}x_1 + \frac{3}{5}x_2 - \frac{3}{10}x_3 = 0 \end{cases}$

(b) $\begin{cases} x_1 + x_2 + x_3 = 1 \\ \frac{7}{10}x_1 + \frac{1}{5}x_2 + \frac{1}{10}x_3 = 0 \\ \frac{1}{10}x_1 + \frac{3}{10}x_2 + \frac{3}{5}x_3 = 0 \end{cases}$

(c) $\begin{cases} x_1 + x_2 + x_3 = 1 \\ -\frac{3}{10}x_1 + \frac{1}{10}x_2 + \frac{1}{5}x_3 = 0 \\ \frac{1}{5}x_1 - \frac{7}{10}x_2 + \frac{1}{2}x_3 = 0 \end{cases}$

(d) $\begin{cases} -\frac{3}{10}x_1 + \frac{1}{5}x_2 + \frac{1}{10}x_3 = 0 \\ \frac{1}{10}x_1 - \frac{3}{10}x_2 + \frac{3}{5}x_3 = 0 \\ \frac{1}{5}x_1 + \frac{1}{2}x_2 - \frac{3}{10}x_3 = 0 \end{cases}$

(e) $\begin{cases} x_1 + x_2 + x_3 = 1 \\ -\frac{3}{10}x_1 + \frac{1}{5}x_2 + \frac{1}{10}x_3 = 0 \\ \frac{1}{10}x_1 - \frac{3}{10}x_2 + \frac{3}{5}x_3 = 0 \end{cases}$

(f) None of the Above

$\frac{7}{10}x + \frac{1}{10}y + \frac{1}{5}z = x$

$\frac{1}{5}x + \frac{3}{10}y + \frac{1}{2}z = y$

$\frac{1}{10}x + \frac{3}{5}y + \frac{3}{10}z = z$

$-\frac{3}{10}x + \frac{1}{10}y + \frac{1}{5}z = 0$

$\frac{1}{5}x - \frac{7}{10}y + \frac{1}{2}z = 0$

$\cancel{\frac{1}{10}x + \frac{3}{5}y - \frac{7}{10}z = 0}$

$x + y + z = 1$

29. Suppose a car rental agency has three locations, numbered 1,2, and 3. A customer may rent a car from any of the three locations and return it to any of the three locations. Records show that cars rented from location 1 are returned to location 2 10% of the time, and to location 3 10% of the time. Cars rented from location 2 are returned to location 1 30% of the time and to location 3 20% of the time. Cars rented from location 3 are equally likely to be returned to locations 2 and 3, and are never returned to location 1. If this system is modelled with a Markov chain, what is the transition matrix?

(a) $P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.33 & 0.33 & 0.34 \end{bmatrix}$

* Go by rows $\longrightarrow$

* Rows add up to 1

(b) $P = \begin{bmatrix} 0.8 & 0.3 & 0 \\ 0.1 & 0.5 & 0.5 \\ 0.1 & 0.1 & 0.5 \end{bmatrix}$

(c) $P = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.3 & -0.5 & 0.1 \\ 0 & 0.5 & -0.5 \end{bmatrix}$

$\boxed{\text{(d)}}$ $P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0 & 0.5 & 0.5 \end{bmatrix}$

(e) There is not enough information to determine the transition matrix.

(f) None of the Above

30. A medical researcher is studying the risk of heart attack in men. She first divides men into three weight categories: thin, normal, and overweight. By studying the male ancestors, sons, and grandsons of these men, the researcher comes up with a transition matrix for one generation. Find the probability that a man who is overweight has a thin grandson. (The first column is thin, middle is normal, and the third column is overweight.)

$$\begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} = \begin{matrix} A \longrightarrow B \\ \text{Father} \rightarrow \text{Son} \end{matrix}$$

squared = $\begin{bmatrix} .25 & .51 & .24 \\ .24 & .52 & .24 \\ .27 & .45 & .28 \end{bmatrix}$ = $A \rightarrow C$ %

Father $\longrightarrow$ Grandson

(a) 0.13

(b) 0.21

(c) 0.22

(d) 0.26

$\boxed{\text{(e)}}$ 0.27

(f) None of the Above

31. Consider a Markov chain with transition matrix $A = \begin{bmatrix} 0.4 & 0.4 & 0.1 & 0.1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0.4 & 0.4 \end{bmatrix}$. Which statement below best describes this Markov chain?

(a) This Markov chain is absorbing because the second row has only 0's and 1's.

(b) This Markov chain is absorbing because it is possible to get from any state to any other state.

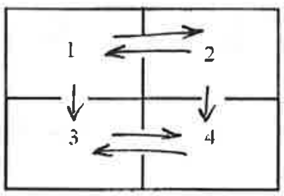
(c) This Markov chain is absorbing because A^3 has only positive entries.

(d) This Markov chain is not absorbing because it is not possible to get from state 2 or state 3 to state 4.

$\boxed{\text{(e)}}$ This Markov chain is not absorbing because it has no absorbing state.

(f) None of the Above

32. A maze for rats has four compartments, as shown below. The rat may move freely between compartments 1 and 2, and may also move freely between compartments 3 and 4. However, the door between compartments 1 and 3 only allows the rat to move from compartment 1 to compartment 3. Likewise, the door between compartments 2 and 4 only allows a rat to move from compartment 2 to compartment 4. The location of the rat is checked every minute and it is found the the rat is equally likely to stay where he is or to move into any available adjacent compartment. What are the absorbing states of this Markov chain?



(a) All states are absorbing.

(b) Only state 3 is absorbing.

(c) Only state 4 is absorbing.

(d) Both states 3 and 4 are absorbing.

$\boxed{\text{(e)}}$ This Markov chain has no absorbing states.

(f) None of the Above.