Mark C. Hughes

CONTACT INFORMATION

Department of Mathematics
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RESEARCH INTERESTS

Low-dimensional topology, including knot theory, braided surfaces, symplectic topology, and categorifications of quantum link invariants. Applications of machine and reinforcement learning to topology.

EMPLOYMENT

Assistant Professor — Aug 2017 to Present
Brigham Young University

Visiting Assistant Professor — Aug 2014 to Aug 2017
Brigham Young University

EDUCATION

Stony Brook University, Stony Brook, NY, USA

Ph.D., Mathematics, 2014

- Thesis: *Braiding non-ribbon surfaces and constructing broken fibrations on smooth 4-manifolds.*
- Advisor: Oleg Viro, Ph.D.

University of Waterloo, Waterloo, ON, Canada

M.Math., Pure Mathematics, 2008

- Thesis: *Branched covering constructions and the symplectic geography problem.*
- Advisor: B. Doug Park, Ph.D.

University of Calgary, Calgary, AB, Canada

B.Sc., Applied Mathematics, 2006

- Minor: Pure Mathematics
- First Class Honours

PUBLICATIONS

Submitted for publication:

1. Jessica Craven, Mark Hughes, Vishnu Jejjala, and Arjun Kar. Illuminating new and known relations between knot invariants. *arXiv preprint arXiv:2211.01404*, 2022.
2. Jessica Craven, Mark Hughes, Vishnu Jejjala, and Arjun Kar. (K)not machine learning. *arXiv preprint arXiv:2201.08846*, 2022.
3. Mark Hughes, Seungwon Kim, and Maggie Miller. Knotted handlebodies in the 4-sphere and 5-ball. *arXiv preprint arXiv:2111.13255*, 2021.
4. Mark Hughes, Seungwon Kim, and Maggie Miller. Band diagrams of immersed surfaces in 4-manifolds. *arXiv preprint arXiv:2108.12794*, 2021.

Accepted or published:

5. Jessica Craven, Mark Hughes, Vishnu Jejjala, and Arjun Kar. Learning knot invariants across dimensions. *SciPost Physics (To Appear)*, 2023.

6. Mark C Hughes. Broken Lefschetz fibrations, branched coverings, and braided surfaces. *Open Book Series*, 5(1):155–184, 2022.

7. Mark C Hughes, Seungwon Kim, and Maggie Miller. Isotopies of surfaces in 4–manifolds via banded unlink diagrams. *Geometry & Topology*, 24(3):1519–1569, 2020.

8. Mark Hughes and Seungwon Kim. Immersed Möbius bands in knot complements. *Algebraic & Geometric Topology*, 20(2):1059–1072, 2020.

9. Mark C Hughes. A neural network approach to predicting and computing knot invariants. *Journal of Knot Theory and Its Ramifications*, 29(03):2050005, 2020.

10. Leslie Colton, Cory Glover, Mark Hughes, and Samantha Sandberg. A Reidemeister type theorem for petal diagrams of knots. *Topology and its Applications*, 267:106896, 2019.

11. Mark C Hughes. Braiding link cobordisms and non-ribbon surfaces. *Algebraic & Geometric Topology*, 15(6):3707–3729, 2016.

12. Mark C Hughes. A note on Khovanov–Rozansky sl_2-homology and ordinary Khovanov homology. *Journal of Knot Theory and its Ramifications*, 23(12):1450057, 2014.

13. Anar Akhmedov, Mark C Hughes, and B Doug Park. Geography of simply connected nonspin symplectic 4-manifolds with positive signature. *Pacific J. Math*, 261(2):257–282, 2013.

AWARDS AND GRANTS

National Science Foundation

- LEAPS-MPS-2213295: *Deep Learning the Knot Landscape*. Principal investigator. Total amount: $249,783. — Sep 2022 to Aug 2024

Department of Mathematics BYU

- Department of Mathematics Distinguished Teaching Award — Dec 2019

College of Physical and Mathematical Sciences BYU

- Mentoring Environment Grant: *Interactions Between Topology and Machine Learning*. Principal investigator. Total amount: $20,000. — Jan 2018 to Dec 2019

Natural Sciences and Engineering Research Council of Canada

- Postgraduate Scholarship — Sep 2008 to Aug 2010
- Canada Graduate Scholarship (*Declined*) — Oct 2008
- Canada Graduate Scholarship — Sep 2006 to Aug 2008
- Undergraduate Student Research Award — May to Aug 2006
- Undergraduate Student Research Award — May to Aug 2005

Academic and Leadership Awards

- President's Graduate Scholarship - *University of Waterloo* — Sep 2006 to Aug 2008
- Student of the Year - *Medicine Hat College* — Apr 2004
- Governor General's Academic Medal - *Government of Canada* — Jun 2000

INVITED TALKS

1. Computer Science for Knotty Math Problems. Dublin Institute for Advanced Studies, Dublin, Ireland. November 2022. *A multi-pronged approach to ML for knot theory.*
2. A Deep-Learning Era of Particle Theory. Mainz Institute for Theoretical Physics, Mainz, Germany. June 2022. *Uncovering the knot landscape via deep learning.*
3. Braids in Low-Dimensional Topology. Institute for Computational and Experimental Research in Mathematics, Providence, Rhode Island. April 2022. *Computational bounds on the band rank.*

4. AMS Special Session on Knot Theory in Dimension Four. Joint Mathematics Meeting. April 2022. *Branched coverings over surface braids and (broken) Lefschetz fibrations on noncompact 4–manifolds.*
5. MAA Special Session on Knots in Dimension 3. Annual Meeting of the Pacific Northwest Section of the MAA, Bellingham, Washington. April 2022. *Nonorientable immersed surfaces in knot complements.*
6. Annual Meeting of the Pacific Northwest Section of the MAA, Bellingham, Washington. April 2022. *Minicourse: Teaching machines to do knot theory.*
7. AMS Special Session on Geometric Topology in the Middle Dimensions. Spring Central Sectional Meeting. March 2022. *Diagrams of immersed surfaces in 4–manifolds.*
8. String Data 2021. University of the Witwatersrand, Johannesburg, South Africa. December 2021. *Using generative adversarial networks to produce knots with specified invariants.*
9. AMS Special Session on Geometric Topology. Fall Central Sectional Meeting. October 2021. *Branched coverings and (broken) Lefschetz fibrations on noncompact 4–manifolds.*
10. Mathematics Department Colloquium. Utah Valley University, Provo, Utah. November 2020. *Training machines to do mathematics.*
11. Virtual Trisectors Seminar. May 2020. *Braided ribbon surfaces and bounds on the band rank.*
12. AMS Special Session on Applications and Computations in Knot Theory. Joint Mathematics Meeting, Denver, Colorado. January 2020. *Reinforcement learning for constructive proofs in topology.*
13. Strings, Geometry, and Data Science Workshop. Simons Center for Geometry and Physics, Stony Brook, New York. January 2020. *Machine and reinforcement learning for constructive proofs in topology.*
14. AMS Special Session on Symplectic and Low Dimensional Topology. Fall Western Sectional Meeting, Riverside, California. November 2019. *Surfaces and isotopies in 4-manifolds via banded unlinks.*
15. Workshop on Unifying 4-Dimensional Knot Theory. Banff International Research Station, Banff, Alberta. November 2019. *Approaches to computational problems from braided surfaces.*
16. AMS Special Session on Invariants of Knots, Links, and Low-dimensional Manifolds. Fall Eastern Sectional Meeting, Binghamton, New York. October 2019. *Surfaces and isotopies in 4-manifolds via banded unlinks.*
17. Handle Friendship Seminar. University of Tokyo, Tokyo, Japan. March 2019. *Unit ribbon surfaces and the Gluck conjecture.*
18. Differential Topology 19. Ritsumeikan University, Tokyo, Japan. March 2019. *Describing surfaces and isotopies in 4-manifolds via banded unlinks.*
19. Topology Seminar. The University of Georgia, Athens, Georgia. November 2018. *Braided surfaces with caps and positive branch points.*
20. Workshop on Math and Machine Learning. Boston College, Boston, Massachusetts. September 2018. *Using deep reinforcement learning for constructive proofs in low-dimensional topology.*
21. Four Dimensional Topology Conference. Osaka City University, Osaka, Japan. September 2018. *Braided surfaces with caps and positive branch points.*
22. Groups-Semigroups-Topology Seminar. University of Nebraska-Lincoln, Lincoln, Nebraska. March 2018. *The immersed cross-cap number of a knot.*
23. Topology Seminar. Georgia Institute of Technology, Atlanta, Georgia. October 2017. *The immersed crosscap number of a knot.*
24. Geometry and Topology Seminar. CUNY Graduate Center, New York, New York. May 2017. *Solving problems in knot theory via reinforcement learning.*
25. Joint Los Angeles Topology Seminar. UCLA, Los Angeles, California. March 2017. *Solving problems in knot theory via reinforcement learning.*
26. Mathematics Department Colloquium. Utah State University, Logan, Utah. December 2016. *Neural knots: a machine learning approach to knot theory.*

27. Topology Seminar. Boston College, Boston, Massachusetts. November 2016. *A neural network approach to computations in the concordance group.*
28. William Rowan Hamilton Geometry and Topology Workshop. Hamilton Mathematics Institute at Trinity College, Dublin, Ireland. August 2016. *Recognizing quasipositive braids and knots.*
29. Mathematics Department Colloquium. University of Saskatchewan, Saskatoon, Saskatchewan. March 2016. *Knot genera and detecting quasipositivity of braids.*
30. Mathematics Department Colloquium. University of Alabama, Tuscaloosa, Alabama. February 2016. *Braid rank and detecting quasipositivity of braids.*
31. Topology Seminar. Syracuse University, Syracuse, New York. November 2015. *Braided cobordisms and the braid rank of a knot.*
32. Topology Seminar. University of Massachusetts Amherst, Amherst, Massachusetts. November 2015. *Braided cobordisms and the braid rank of a knot.*
33. Max Dehn Seminar. University of Utah, Salt Lake City, Utah. October 2015. *Braided cobordisms and the braid rank of a knot.*
34. Differential Geometry and Symplectic Topology Seminar. University of Minnesota, Minneapolis, Minnesota. February 2014. *Constructions of broken fibrations on 4-manifolds.*
35. Topology Seminar. Georgia Institute of Technology, Atlanta, Georgia. January 2014. *A new bound on the ribbon genus of knots via braided surfaces.*

CONTRIBUTED TALKS

1. AMS-SIGMAA MCST Special Session on Math Circle Activities as a Gateway into Mathematics. Joint Mathematics Meeting, Boston, Massachusetts. January 2023. *Modeling complex behavior from simple rules: Cellular automata for Math Circles.*
2. 39th Annual Workshop in Geometric Topology. June 2022. *Branched coverings over surface braids and (broken) Lefschetz fibrations on non-compact 4-manifolds.*
3. AMS Special Session on Theoretical and Applied perspectives in Machine Learning. Fall Western Sectional Meeting. October 2021. *Using deep learning to generate knots with prescribed invariants.*
4. AMS Special Session on Developments in Knot Theory and Low-dimensional Topology. Fall Central Sectional Meeting. October 2021. *Using deep learning to generate knots with prescribed invariants.*
5. MAA Intermountain Sectional Meeting. March 2021. *Designing a computational linear algebra lab course using Google Colab.*
6. MAA Contributed Paper Session on Innovative and Effective Ways to Teach Linear Algebra. Joint Mathematics Meeting, Denver, Colorado. January 2020. *Designing a computational linear algebra lab course using Google Colab.*
7. AMS Contributed Paper Session on General Topology, Algebraic Topology, and Topology of Manifolds. Joint Mathematics Meeting, Baltimore, Maryland. January 2019. *Immersed Möbius bands in knot complements and representatives of $\mathbb{Z}_2$-homology classes.*
8. AMS Contributed Paper Session on Knots and Diagram Categories. Joint Mathematics Meeting, San Diego, California. January 2018. *The immersed cross-cap number of a knot.*
9. MAA Session on Math Circle Topics with Visual or Kinesthetic Components. Joint Mathematics Meeting, San Diego, California. January 2018. *Knotted mathematics for elementary-aged students.*
10. MAA Session on Unexpected Topics for a Math Circle. Joint Mathematics Meeting, Atlanta, Georgia. January 2017. *Complex behavior from simple rules: cellular automata for Math Circles.*
11. AMS Contributed Paper Session on Topology and Manifolds. Joint Mathematics Meeting, Atlanta, Georgia. January 2017. *A neural network approach to computing knot invariants.*
12. AMS Session on Topology and Knot Theory. Joint Mathematics Meetings, Seattle, Washington. January 2016. *Braided cobordisms and the braid rank of a knot.*
13. MAA Session on Innovative and Effective Ways to Teach Linear Algebra. Joint Mathematics Meetings, Seattle, Washington. January 2016. *Exploring linear algebra with technology while being crunched for time.*

14. Knots in Washington XLI. George Washington University, Washington DC. December 2015. *Braided cobordisms and the braid rank of a knot*
15. AMS Special Session on Geometry of Groups, Surfaces and 3-manifolds. Fall Eastern Sectional Meeting, Rutgers University, New Brunswick, New Jersey. November 2015. *Braided cobordisms and the braid rank of a knot.*
16. AMS Special Session on Knots and 3-Manifolds. Spring Western Sectional Meeting, University of Nevada, Las Vegas, Nevada. April 2015. *Braiding knot cobordisms.*
17. Knots in Washington XXXVII. George Washington University, Washington DC. January 2014. *Comparing slice and ribbon genera via braided surfaces.*
18. Knots in Washington XXXV. George Washington University, Washington DC. December 2012. *Alexander and Markov type theorems for link cobordisms.*
19. Graduate Student Topology Conference. Indiana University, Bloomington, Indiana. April 2012. *Braided link cobordisms.*

CONFERENCES AND WORKSHOPS ORGANIZED

1. *AMS Special Session on 4-Dimensional Topology.* Co-organizer with Maggie Miller and Patrick Naylor. Fall Western Sectional Meeting. October 2022.
2. *Applying Machine Learning to Mathematics Mini-Workshop.* Co-organizer with Elisenda Grigsby, Maurizio Parton, and Radmila Sazdanovic. Institute for Computational and Experimental Research in Mathematics. April 2022.
3. *AMS Special Session on Knotted Surfaces and Concordances.* Co-organizer with Jeffrey Meier and Maggie Miller. Fall Western Sectional Meeting. October 2020.
4. *Moab Topology Conference 2019.* Co-organizer with Nathan Geer. Moab, Utah. May 2019.
5. *Moab Topology Conference 2015.* Co-organizer with Nathan Geer and Jessica Purcell. Moab, Utah. May 2015.

STUDENTS SUPERVISED

Undergraduate Mentored Research Advisor

1. Alexander Bystrom (Jan 2023 – Present): *Minimizing the length of band decompositions of braids via reinforcement learning.*
2. Adam Call (May 2022 – Present): *Applying genetic algorithms to knot theory.*
3. Nathaniel Stevenson (Sep 2021 – Present): *Knot representations in open book decompositions of 3 – manifolds.*
4. Timothy Keith (May 2021 – Present): *Learning representations of knot spaces using variational autoencoders.*
5. Dylan Skinner (Apr 2021 – Present): *Finding minimal genus slice surfaces via deep reinforcement learning.*
6. Jason Garcia (May 2022 – Oct 2022): *Applying reinforcement learning to knot theory.*
7. Curtis Evans (May 2022 – Sep 2022): *Applying reinforcement learning to knot theory.*
8. Jackson Switzer (Apr 2021 – Mar 2022): *Finding minimal genus slice surfaces via deep reinforcement learning.*
9. Neil Thompson (Jun 2021 – Sep 2021): *Predicting knot invariants via machine learning.*
10. Dahlia Maxwell (Jul 2020 – Aug 2021): *Slice genus computations for low-crossing knots; invariants of knots from petal diagrams.*
11. Tanner Osburn (Sep 2020 – Jun 2021): *Slice genus computations for low-crossing knots.*
12. Seth Hall (Sep 2020 – Jun 2021): *Slice genus computations for low-crossing knots.*
13. Amy Eubanks (Apr 2020 – Apr 2021): *Using reinforcement learning to find minimal genus slice surfaces and minimizing Hurewitz moves.*
14. Jared Slone (Sep 2019 – Apr 2021): *Using reinforcement learning to find minimal genus slice surfaces and minimizing Hurewitz moves.*
15. Brevan Ellefsen (Jun 2019 – Jan 2021): *Invariants of knots from petal diagrams; characterizing petal diagrams of torus links.*
16. Andrea Barton (May 2019 – May 2020): *Invariants of knots from petal diagrams; characterizing petal diagrams of torus links.*
17. Thomas Liddle (Jan 2019 – Mar 2020): *Using reinforcement learning to find minimal*

genus slice surfaces and minimizing Hurewitz moves.

18. Jamison Moody (Aug 2018 – Jan 2020): *Using reinforcement learning for solving diagrammatic problems in knot theory.*
19. Samantha Sandberg (Jan 2018 – Aug 2019): *A Reidemeister type theorem for petal diagrams of knots.*
20. Jonathan Edevold (Apr 2018 – Apr 2019): *Using reinforcement learning for solving diagrammatic problems in knot theory.*
21. Cory Glover (Jan 2018 – Apr 2019): *A Reidemeister type theorem for petal diagrams of knots.*
22. Leslie Colton (May 2017 – Apr 2019): *Bounds on the Turaev genus of knots, a Reidemeister type theorem for petal diagrams of knots.*
23. Spencer Reschke (Feb 2017 – Apr 2019): *Data mining the Math Feeds app, using reinforcement learning to find efficient isoperimetric reductions of matrices, using reinforcement learning to find minimal genus slice surfaces.*
24. Bryce Pierson (May 2017 – Mar 2019): *Data mining the Math Feeds app, using reinforcement learning to find efficient isoperimetric reductions of matrices, constructing group representations via neural networks.*
25. Jeongwoo Kim (Jan 2018 – Sep 2018): *Using reinforcement learning to find efficient isoperimetric reductions of matrices.*
26. Kaden Barlow (Jan 2018 – Apr 2018): *Data mining the Math Feeds app.*
27. Tyler Jones (Jan 2018 – Apr 2018): *Using reinforcement learning to find efficient isoperimetric reductions of matrices.*
28. Seong-Eun Cho (Feb 2017 – Apr 2018): *Data mining the Math Feeds app, using reinforcement learning to find efficient isoperimetric reductions of matrices.*
29. Matthew Oehler (Jan 2016 – Apr 2018): *Q-learning for asymmetric two player games, data mining the Math Feeds app.*
30. Zachary Taylor (Jan 2016 – Apr 2018): *Q-learning for asymmetric two player games, data mining the Math Feeds app.*
31. Brinley Poulsen (Jan 2017 – Apr 2017): *Data mining the Math Feeds app.*
32. Joshua Wilson (Oct 2016 – Apr 2017): *Q-learning for asymmetric two player games, data mining the Math Feeds app.*
33. Rachel Newell (Jan 2016 – Apr 2017): *Using neural networks for artwork classification, data mining the Math Feeds app.*
34. Gabriel Bradford (Jan 2016 – Feb 2017): *Q-learning for asymmetric two player games.*
35. Zach Horton (Jan 2016 – Feb 2017): *Q-learning for asymmetric two player games.*
36. McKay Kerksiek (Jan 2016 – Feb 2017): *Using neural networks for artwork classification.*
37. Kolten Pearson (Jan 2016 – Feb 2017): *Using neural networks for artwork classification.*
38. Megan Searles (Jan 2016 – Feb 2017): *Using neural networks for artwork classification.*

Undergraduate Honors Committee Chair

1. Dylan Skinner (Honors advisor Apr 2021 – Present).

Graduate Committee Chair

1. Dahlia Maxwell (M.S. advisor Sep 2021 – Apr 2022).
2. Jason Gardiner (M.S. thesis defended Jul 2021). Thesis title: *Petal diagrams and Seifert surfaces.*
3. Justin Meiners (M.S. thesis defended Mar 2021). Thesis title: *Computing the rank of braids.*

Graduate Committee Member

1. Dahlia Maxwell (M.S. committee member Apr 2022 – Present)
2. Daniel Jensen (M.S. committee member Mar 2021 – Present)
3. Xueming Hui (Ph.D. committee member Jun 2020 – Present)
4. Tyler Moncur (M.S. thesis defended Jul 2020). Thesis title: *Optimal Learning Rates for*

Neural Networks.

5. Michael Andersen (Ph.D. dissertation defended Nov 2019). Dissertation title: *Almost Homeomorphisms and Inscrutability.*
6. Tyler Hills (Ph.D. dissertation defended Nov 2019). Dissertation title: *An Equivalence of Shape and Deck Groups; Further Classification of Sharkovskii Groups.*
7. Benjamin Schoonmaker (Ph.D. dissertation defended Oct 2018). Dissertation title: *Clean Indices of Common Rings.*

TEACHING EXPERIENCE

Instructor - Brigham Young University

• MATH 113 - Calculus II	2014, 2017, 2019, 2020
• MATH 213 - Elementary Linear Algebra	2019 to 2022
• MATH 215 - Computational Linear Algebra	2021 to 2022
• MATH 313 - Elementary Linear Algebra	2014 to 2019
• MATH 290 - Fundamentals of Mathematics	2016, 2018, 2021, 2022
• MATH 451 - Introduction to Topology	2021 to 2023
• MATH 565 - Differential Geometry	2019
• MATH 655 - Differential Topology	2015, 2017, 2020, 2022

Instructor - Stony Brook University

• MAP 103 - Proficiency Algebra	2010, 2013
• MAT 118 - Mathematical Thinking	2010, 2011, 2012
• MAT 127 - Calculus C	2012, 2013

Teaching Assistant - Stony Brook University

• MAT 123 - Introduction to Calculus	2012
• MAT 125 - Calculus A	2011
• MAT 126 - Calculus B	2010
• MAT 132 - Calculus II	2011, 2013

Teaching Assistant - University of Waterloo

• Calculus and Linear Algebra courses	2006 to 2008

SERVICE

BYU Data Science Curriculum Committee Member	Aug 2022 to Present
BYU Math Department Hiring Committee Member	Aug 2022 to Present
BYU Math Department Undergraduate Committee Member	Aug 2022 to Present
BYU Data Science Club Faculty Advisor	Sep 2017 to Present
Intermountain Mathematics Competition Co-organizer	Nov 2021
BYU Math Department Competitions Committee Member	Aug 2021 to July 2022
Math 213/313 Course Coordinator	May 2018 to July 2022
Math 215 Course Coordinator	May 2018 to July 2022
Math 213 and 215 Course Developer	May 2018 to Apr 2021
BYU Math Department Public Relations Committee Member	May 2018 to Apr 2019
BYU Math Department Writing Group Coordinator	Sep 2017 to Apr 2019
BYU Calculus Committee Member	May 2017 to May 2018
BYU Math Department TA Training Workshop Organizer	Aug 2017
Math 313 Course Coordinator	Sep 2016 to Apr 2017
BYU Data Science Club Faculty Co-advisor	Jan 2016 to Apr 2016

Kindergarten to 3rd Grade Math Circle Director	Sep 2015 to Apr 2016
BYU Math Department Colloquium Co-organizer	Sep 2015 to Apr 2016
BYU Math Department Curriculum Committee Member	Sep 2014 to Apr 2015