• **Welington Cordeiro** (IMPA)

**Title:** CW-expansive flows and entropy

**Abstract:** The study of expansive dynamical systems for more general dynamics attracts a lot of attention by many researchers. Indeed, expansive homeomorphisms are classical in the literature. In 1972, Bowen and Walters defined expansivity for flows and they proved that the suspension flows of an expansive homeomorphism is expansive. In 1993, Hisao Kato defined that a homeomorphism $f : X \to X$ is continuum wise expansive if there exists $\epsilon > 0$ such that if $A \subset X$ is a non-trivial compact and connected set then there is $n \in \mathbb{Z}$ such that $\text{diam}(f^n(A)) > \epsilon$. In this work, we try to define the continuum wise expansive for flows, and we’ll prove that continuum wise expansive flows have positive entropy and the suspension flow of a homeomorphism is CW-expansive if, and only if, the homeomorphism is CW-expansive.

• **Enzo Fuentes** (PUCV)

**Title:** Generic Hölder foliations

**Abstract:** In this work, we study an example of a space of foliations for $M = [0,1]^2$ (with some topology) that generically are not absolutely continuous, furthermore, the conditional measures defined by Rokhlin disintegration are Dirac measures. These foliations are motivated by the foliations that appear in hyperbolic systems and partially hyperbolic systems.

*Date:* May 31, 2017.
• **Andrey Gogolev** (Binghamton University, SUNY)

Title: Numerics for strong unstable manifolds and u-measures on the 3-torus  

Abstract: This is a preliminary report on a joint work with A. Kolmogorov (SUNY, Physics) and I. Maimon (SUNY 17). Consider a hyperbolic automorphism $A : T^3 \rightarrow T^3$ with real spectrum whose 2-dimensional unstable distribution splits into weak and strong sub-bundles. We unfold $A$ into an analytic family $f_\epsilon$ and perform a very accurate numeric calculation of the strong unstable manifolds $W_{f_\epsilon}^{uu}(p)$, where $p$ is a fixed point. Our calculation indicate that the strong unstable manifold stays dense in the 3-torus. Further we amend our calculation of $W_{f_\epsilon}^{uu}(p)$ with a numeric calculation of the strong unstable Jacobian to obtain approximations of the (strong) u-measure associated to $W_{f_\epsilon}^{uu}(p)$. Our numerics indicate that the u-measure coincides with the SRB measure.

• **Pierre-Antoine Guihéneuf** (Université Paris 6/IMJ-PRG)

Title: Local and global behaviours of expanding maps’ discretizations  

Abstract: Given a $C^2$ expanding map $f$ of the circle (i.e. $f' > 1$), a classical theorem states that the pushforwards of Lebesgue measure converge towards the so called SRB measure when $n$ goes to infinity. Following a question of (among others) Lanford, one can ask whether this result persists when $f$ is replaced by a $N$-digits discretization of $f$. We present results expressing to what extend this holds for a ”small” number of iterations $n$ (work in progress with M. Monge).

• **Baolin He** (Shanghai Normal University)

Title: Entropy on the line  

Abstract: In this paper, we study the continuity of entropy map on line w.r.t. the strong topology on the space of diffeomorphisms on line ($\text{Diff}_s^r(\mathbb{R})$). We prove that the entropy map is continuous on an open and dense subset of $\text{Diff}_s^r(\mathbb{R})$, $r = 1, 2, 3, 4, \ldots$, and we construct two examples: 1. there exists $C^\infty$ diffeomorphism $f$ such that the entropy map is neither upper-semicontinuous or lower-semicontinuous at $f$ ; 2. there exists an open subset $U$ of $\text{Diff}_s^r(\mathbb{R})$ such that the entropy map is not continuous at every $f \in U$.  
• Gabriella Keszthelyi (Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences)

Title: Equi-topological entropy curves for skew tent maps in the square

Abstract: We consider skew tent maps $T_{\alpha,\beta}(x)$ such that $(\alpha, \beta) \in [0,1]^2$ is the turning point of $T_{\alpha,\beta}$, that is, $T_{\alpha,\beta} = \frac{\beta}{\alpha} x$ for $0 \leq x \leq \alpha$ and $T_{\alpha,\beta}(x) = \frac{\beta}{1-\alpha}(1-x)$ for $\alpha < x \leq 1$. We denote by $M = K(\alpha, \beta)$ the kneading sequence of $T_{\alpha,\beta}$ and by $h(\alpha, \beta)$ its topological entropy. For a given kneading sequence $M$ we consider equi-kneading, (or equi-topological entropy, or isentrope) curves $(\alpha, \varphi_M(\alpha))$ such that $K(\alpha, \varphi_M(\alpha)) = M$. To study the behavior of these curves an auxiliary function $\Theta_M(\alpha, \beta)$ is introduced. For this function $\Theta_M(\alpha, \varphi_M(\alpha)) = 0$, but it may happen that for some kneading sequences $\Theta_M(\alpha, \beta) = 0$ for some $\beta < \varphi_M(\alpha)$ with $(\alpha, \beta)$ still in the dynamically interesting quarter of the unit square. Using $\Theta_M$ we show that the curves $(\alpha, \varphi_M(\alpha))$ hit the diagonal $\{(\beta, \beta) : 0.5 < \beta < 1\}$ almost perpendicularly if $(\beta, \beta)$ is close to $(1,1)$. Answering a question asked by M. Misiurewicz at a conference we show that these curves are not necessarily exactly orthogonal to the diagonal, for example for $M = RLLRC$ the curve $(\alpha, \varphi_M(\alpha))$ is not orthogonal to the diagonal. On the other hand, for $M = RLC$ it is.

With different parametrization properties of equi-kneading maps for skew tent maps were considered by J.C. Marcuard, M. Misiurewicz and E. Visinescu.

• Gemma Mason (The University of Auckland)

Title: A Heterodimensional Cycle in a 4D Atri Model

Abstract: We investigate a four-dimensional ordinary differential equation model for intracellular calcium dynamics. This model exhibits a heterodimensional cycle, which is a heteroclinic connection between two saddle-periodic orbits whose corresponding stable manifolds are of different dimensions. Heterodimensional cycles are associated with new types of chaotic dynamics in diffeomorphisms of three or more dimensions, and hence in differential equation systems of four or more dimensions. In particular, it has been shown by Bonatti and Diaz that, given a $C^1$ diffeomorphism containing a codimension-one heterodimensional cycle, there is an arbitrarily $C^1$-close set of diffeomorphisms that have robust codimension-one heterodimensional cycles. We wish to examine how the heterodimensional cycle in the flow of this differential equation affects the nearby dynamics. To observe the behavior of this practical example, we require
computational tools and visualization techniques for invariant manifolds of periodic orbits in four dimensions. A three-dimensional Poincaré section provides an overview of the models behavior. Projections of the four-dimensional flow into three dimensions are then used to explain the behavior of objects in the Poincaré section in more depth.

• **Gabriel Ponce** (State University of Campinas - UNICAMP)

Title: The atomic nightmare lays between flow ergodicity and discrete ergodicity

**Abstract:** The dynamics of a flow is, a priori, very different from the dynamics of its time-one map. It is not at all expected that an ergodic probability for the time-one map to be ergodic (in particular invariant) for the flow. It is actually very natural to ask what are the characteristics of ergodic measures which are invariant by the time-1 map of a flow but not flow invariant. That is, what are the obstructions laying between the flow ergodicity and the discrete time ergodicity? We show that if an ergodic measure for the time-one map is not ”too much pathological in the orbits” (i.e. it does not have atomic disintegration on the foliation given by the orbits), then the measure is ergodic for the flow, in particular invariant. As a corollary we also obtain an interesting conjugacy rigidity result.

• **Raquel Ribeiro** (Universidade de São Paulo)

Title: Shadowing for Discontinuous Linear Maps.

**Abstract:** Let $f$ be a map in a compact metric space $X$. A sequence of points $\{x_i\}_{i \in \mathbb{Z}}$ is a $\alpha$-pseudo-orbit $(\alpha > 0)$ of $f$ if

$$d(f(x_i), x_{i+1}) \leq \alpha, \text{ for all } i \in \mathbb{Z}.$$ $$d(f^i(x), x_i) \leq \epsilon \text{ for all } i \in \mathbb{Z}.$$ $f$ has the shadowing property if, given $\epsilon > 0$ there exists $\alpha > 0$ such that for every $\alpha$-pseudo-orbit $\{x_i\}_{i \in \mathbb{Z}}$, there is a point $x \in X$ satisfying, $d(f^i(x), x_i) \leq \epsilon$ for all $i \in \mathbb{Z}$.

The shadowing property was introduced by Anosov, and it is closely related to the stability property of dynamical systems. In 1988, the authors Coven, Kan, Yorke studied the shadowing property for the family of tent maps, that is, the linear maps $f_s : [0, 2] \rightarrow [0, 2], \sqrt{2} \leq s \leq 2$ defined as follows:

$$f_s(x) = \begin{cases} sx & 0 \leq x \leq 1 \\ s(2-x) & 1 \leq x \leq 2. \end{cases}$$
The authors have provided sufficient and necessary conditions for a tent map to have the shadowing property. In 1991, this result was generalized to continuous piecewise linear maps.

We study discontinuous piecewise linear maps of a compact interval and we prove that the shadowing property is completely determined by the dynamics of the points at which such a map is not monotonous, or is discontinuous. This result generalizes the above.

• Juan David Rojas Gacha (IMPA)

Title: Generic families exhibiting infinitely many non-uniform hyperbolic attractors for a set of total measure of parameter.

Abstract: We show the existence of a Baire residual set of families of local diffeomorphisms on a two-dimensional manifold, displaying infinitely many Hénon-like strange attractors for Lebesgue almost every parameter. This result gives a partial answer (in the context of local diffeomorphisms) to a question asked by Colli, in the late nineties.

• Jaqueline Siqueira (PUC-Rio)

Title: On statistical properties for equilibrium states of partially hyperbolic horseshoes

Abstract: We prove uniqueness of equilibrium states of partially hyperbolic horseshoes associated to Hölder continuous potentials with small variation. (Joint work with Isabel Rios). We derive some statistical properties for the unique equilibrium state. For this we define a projection map associated to the horseshoe and prove a spectral gap for its transfer operator acting on the space of Hölder continuous observables. From this we deduce an exponential decay of correlations and a central limit theorem. Finally, we extend these results to the horseshoe. (Joint work with Vanessa Ramos).

• Luciana Salgado (UFRJ / UFBA)

Title: Dominated splitting for exterior powers and singular hyperbolicity

Abstract: We show that an invariant splitting for the tangent map to a smooth flow over a compact invariant subset is dominated if, and only if, the exterior power of the tangent map admits an invariant dominated splitting. For a differentiable vector field $X$ on a 3-manifold, we obtain singular hyperbolicity using only the tangent map $DX$ of $X$ and a family
of indefinite and non-degenerate quadratic forms without using the associated flow and its derivative. As a consequence, we show the existence of adapted metrics for singular hyperbolic sets for three-dimensional vector fields.
• **Diego Sanhueza** (Universidade Federal do Rio de Janeiro)

**Title:** Specification and almost specification properties

**Abstract:** Our goal is to study some topological and ergodic properties for dynamical systems, in both discrete and continuous time, satisfying a property called the (almost) specification property, although we will be focused in getting about entropy results from generic points viewpoint. For instance, Pfister and Sullivan have shown that if a dynamical system has the almost specification property, then for any invariant measure the topological entropy of the generic points and metric entropy of the system regarding the measure are equals.

• **Shirou Wang** (Chinese Academy of Sciences)

**Title:** Ruelle inequality of folding type for $C^{r+1}$ maps (joint work with Gang Liao).

**Abstract:** Let $f$ be a differentiable map on a compact Riemmanian manifold $M$ without boundary and $\mu$ an $f$-invariant Borel probability measure. According to the well known Margulis-Ruelle inequality, the metric entropy $h_\mu(f)$ is bounded by the sum of positive Lyapunov exponents from above:

$$h_\mu(f) = \int_M \sum_{\lambda_i(f,x) > 0} \lambda_i(f,x) d\mu(x). \quad (1)$$

For studying the positivity of entropy production in nonequilibrium statistical mechanics, Ruelle [J. Stat. Phys. 85(1/2), 1C23,1996.] conjectured, in conformity with the backward process of (1), the following inequality of folding type:

$$h_\mu(f) \leq F_\mu(f) - \int_M \sum_{\lambda_i(f,x) < 0} \lambda_i(f,x) d\mu(x), \quad (2)$$

where $F_\mu(f)$ denotes the folding entropy of $\mu$, i.e., $F_\mu(f) = H_\mu(\epsilon | f^{-1}\epsilon)$ with $\epsilon$ being the partition of $M$ into single points and $H_\mu(\epsilon | f^{-1}\epsilon)$ the conditional entropy of $\mu$ related to $f^{-1}\epsilon$. Because of the non-invertibility, in (2), apart from Lyapunov exponents which indicate the divergence rate on tangent spaces, the folding of the map also contributes entropy when one selects uncertain preimages to be deterministic.

In the setup of local diffeomorphisms, as pointed out by Ruelle, one can deduce (2). For general cases, the existence of degenerate points brings about essential difficulties. Firstly, the diameter of regular neighborhoods on which $f$ behaves like diffeomorphisms has no positive lower bound;
secondly, the number of pre-images may grow to the infinity; thirdly, a probably most essential obstruction in the estimate of partition entropy is that the images of a zero measured degenerate set may have positive measure (hence the zero measured degenerate set also needs to be analyzed).

In 2003, P. Liu[CMP.240(3)] provided the proof of (2) for $C^{1+\alpha}$ maps with polynomial-like degeneracy, integrable preimages and zero measured images of degenerate sets. In this paper, we establish (2), removing conditions on the degenerate sets by constructing a new partition to estimate the degeneracy parts corresponding to the three difficulties mentioned above and address (2) for all $C^{1+\alpha}$ maps.

**Theorem.** Let \( f \) be a $C^{1+\alpha}$ map on a compact Riemannian manifold \( M \) without boundary and \( \mu \) an \( f \)-invariant Borel probability measure. Then the inequality (2) holds true.

**Theorem.** Let \( f \) be a $C^{1+\alpha}$ map on a compact Riemannian manifold \( M \) without boundary and \( \mu \) an \( f \)-invariant Borel probability measure. Then the inequality (2) holds true.

**• Xiaodong Wang** (Shanghai Jiao Tong University)

**Title:** Hyperbolicity versus weak periodic orbits inside homoclinic classes

**Abstract:** We prove that, for $C^1$-generic diffeomorphisms, if the periodic orbits contained in a homoclinic class $H(p)$ have all their Lyapunov exponents bounded away from 0, then $H(p)$ must be (uniformly) hyperbolic. This is in sprit of the works of the stability conjecture, but with a significant difference that the homoclinic class $H(p)$ is not known isolated in advance, hence the weak periodic orbits created by perturbations near the homoclinic class have to be guaranteed strictly inside the homoclinic class. In this sense the problem is of an intrinsic nature, and the classical proof of the stability conjecture does not pass through.