Math 495R Homework 13

In this lab you will write functions to row reduce a matrix to echelon form. Recall that in previous labs we have represented a matrix as an array of arrays, where each inner array represents a **row** of the matrix. Note that this lab only require you to reduce to echelon form, not reduced echelon form.

(1) In this first step you will write a function to handle the necessary swapping operations in a row reduction. Your function should take an $m \times n$ matrix $A = [a_{i,j}]$ and a specified row and column (indicated by indexes i and j respectively). If the entry $a_{i,j}$ is not 0, your function should return A unaltered. Otherwise, your function should step through the entries of the j-th column. Once it finds an entry $a_{k,j} \neq 0$ with $i \leq k \leq m$, the function should the return the matrix A' with rows i and k swapped.

For instance, if $i = j = 1$ and $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, the function would work down the first column, find that a non-zero entry occurs in row 2, then return the matrix $A' = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ where the first and second rows have been swapped.

If $i = 2, j = 3$ and $A = \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$, the function would swap rows 2 and 3.

(2) In this second step you will write a function to handle the necessary row addition operations in a row reduction. Your function should take an $n \times n$ square matrix $A = [a_{i,j}]$ and a specified row and column (indicated by indexes i and j respectively). If the entry $a_{i,j}$ is not 0, your function should iterate through the rows r_k of A with index $k > i$. At each step, replace the row r_k with the row $r_k - \frac{a_{k,j}}{a_{i,j}} r_i$.

For instance, if $i = j = 1$ and $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$, the function would replace the second row with $r_2 - r_1$, and replace the third row with $r_3 - \frac{1}{2} r_1$, resulting in the matrix $A' = \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 1 & 1 \end{bmatrix}$.

(3) Write a function that row-reduces a given matrix and returns the matrix in echelon form. Your function only needs one `for`-loop, where the variable is the column index j. The row index i should be initialized to 1 and step along inside the same loop as you locate pivot positions. Specifically, if a column has no pivot position, j should step forward, but i should not. You may want to modify one of your functions from problem 1 or problem 2 to give a signal if this has occurred.

Test your program on the following matrices:

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 1 & 2 \end{bmatrix}.$$