Math 495R Homework 15

(1) Project Euler, problem 7.
(2) Write a recursive function `gcd` that takes two integers a and b and returns their greatest common divisor. Your function should return an error message if a and b are both zero.
(3) Write a recursive function `xgcd(a, b)` that performs the extended Euclidean algorithm on two integers a and b and returns integers `(d, x, y)` where d is the gcd and x and y are integers such that $d = ax + by$.

    Recall that to write a recursive function we follow these steps:
    (a) Handle the base case. If we are not in the base case, then
    (b) Move one step towards the base case,
    (c) Assume the function will work as intended on the simpler case, and then use it to complete the current case.

    For the `xgcd` function the base case is when $b = 0$, $x = 1$, and $y = 0$. We step towards the base case by using the division algorithm to find q and r so that $a = qb+r$, with $0 \leq r < b$. We then compute `(d, x, y) = xgcd(b, r)`, assuming the function will work as it is supposed to.

    Now since $d = \gcd(b, r) = \gcd(a, b)$, if we have $d = xb + yr$ written as a linear combination of b and r, and we know that $r = a - qb$, we can rearrange to find that $d = ay + b(x - qy)$ as a linear combination of a and b. Thus, we can return the triple `(d, y, (x-qy))`.