Math 495R Homework 18

(1) The International Standard Book Number (ISBN) is an example of an error detecting code. It is a 10-digit (pre-2007) or 13-digit (post-2006) codeword assigned to a published book. For a 10-digit ISBN $a_1 a_2 \ldots a_{10}$, the last digit is chosen so that the digits satisfy

$$\sum_{k=1}^{10} k \cdot a_k \equiv 0 \pmod{11}.$$

For a 13-digit ISBN $a_1 a_2 \ldots a_{13}$, the last digit is chosen so that the digits satisfy

$$a_1 + 3a_2 + a_3 + 3a_4 + a_5 + 3a_6 + a_7 + 3a_8 + a_9 + 3a_{10} + a_{11} + 3a_{12} + a_{13} \equiv 0 \pmod{10}.$$

Write functions `isbn10check` and `isbn13check` that check whether an ISBN is valid. Test these functions on the ISBNs of several books. Note that for ISBN-10, if the check digit is 10 it will appear as an X.

(2) The book Auxiliary Polynomials in Number Theory by David Masser, used in Math 687R in fall 2018, makes the claim that "We may note that the equation $y^2 = x^5 + 3x^4 + x^3 + 6x^2 + 6$ over $\mathbb{Z}_7$ has no solution in $\mathbb{Z}_7^2$." (He uses $\mathbb{F}_7$ in place of $\mathbb{Z}_7$, but it means the same thing.) Write code that confirms this assertion.

(3) Problem 6.13 of Masser's book asks whether there is a polynomial $Q(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ with $a, b, c, d, e, f \in \mathbb{Z}_{11}$ such that there are no solutions $(x, y) \in \mathbb{Z}_{11}^2$ with $y^2 = Q(x)$. Write code that finds such a polynomial.