Math 495R Homework 18

(1) The International Standard Book Number (ISBN) is an example of an error detecting code. It is a 10-digit (pre-2007) or 13-digit (post-2006) codeword assigned to a published book. For a 10-digit ISBN $a_1a_2\ldots a_{10}$, the last digit is chosen so that the digits satisfy
\[
\sum_{k=1}^{10} k \cdot a_k \equiv 0 \pmod{11}.
\]

For a 13-digit ISBN $a_1a_2\ldots a_{13}$, the last digit is chosen so that the digits satisfy
\[
a_1 + 3a_2 + a_3 + 3a_4 + a_5 + 3a_6 + a_7 + 3a_8 + a_9 + 3a_{10} + a_{11} + 3a_{12} + a_{13} \equiv 0 \pmod{10}.
\]

Write functions isbn10check and isbn13check that check whether an ISBN is valid. Test these functions on the ISBNs of several books. Note that for ISBN-10, if the check digit is 10 it will appear as an $X$.

(2) The book Auxiliary Polynomials in Number Theory by David Masser, used in Math 687R in fall 2018, makes the claim that “We may note that the equation $y^2 = x^5 + 3x^4 + x^3 + 6x^2 + 6$ over $\mathbb{Z}_7$ has no solution in $\mathbb{Z}_7^2$. ” (He uses $\mathbb{F}_7$ in place of $\mathbb{Z}_7$, but it means the same thing.) Write code that confirms this assertion.

(3) Problem 6.13 of Masser’s book asks whether there is a polynomial $Q(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ with $a, b, c, d, e, f \in \mathbb{Z}_{11}$ such that there are no solutions $(x, y) \in \mathbb{Z}_{11}^2$ with $y^2 = Q(x)$. Write code that finds such a polynomial.