Math 495R Homework 17

(1) Write a function that takes a matrix and returns the *reduced* row echelon form of that matrix. To do so, you should modify the function you wrote in lab 13 to row-reduce a matrix to echelon form. Recall that a matrix is in reduced echelon form if
  (a) It is in echelon form,
  (b) The entry in each pivot position is a 1,
  (c) Each pivot position is the only non-zero entry in its column.

(2) Write a function that takes an $m \times n$ matrix and augments it with the $m \times m$ identity matrix. For instance, given the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$, the function would return the matrix $\begin{bmatrix} 2 & 2 & 2 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{bmatrix}$.

(3) Write a function that takes a matrix and attempts to invert it. Recall that that the algorithm to invert an $n \times n$ matrix is as follows:
  - Augment the matrix with the $n \times n$ identity matrix.
  - Row reduce the augmented matrix to reduced echelon form.
  - If the first $n$ columns form an identity matrix, then the last $n$ columns form the inverse of the matrix.

  If the matrix is not invertible, you should raise exceptions to notify the user. Exceptions are used to notify the user that a function was used improperly. Unless appropriately caught, an exception will immediately terminate not only the current function, but also every function above it in the stack. So for instance if function $A$ calls function $B$ which calls function $C$, and $C$ raises an exception, then all three functions will terminate without returning a value, and the exception message will be printed.

  If the original matrix is not square, raise an exception using the following code:
`raise Exception('Input matrix is not square!')`
If the matrix is square but not invertible, raise an exception using the following code:
`raise Exception('Input matrix is not invertible!')`