(1) Recall that in Homework 15, you wrote a function \( \text{xgcd}(a, n) \) that returns integers\( d, x, y \) where \( d \) is the greatest common divisor of \( a \) and \( n \) and satisfies \( d = ax + ny \). If we work in \( \mathbb{Z}_n \), this equation becomes \( d \equiv ax \pmod{n} \), or \( d = a \cdot x \). If \( a \) and \( n \) are relatively prime, so that \( d = 1 \), we find that our \( x \) satisfies \( ax \equiv 1 \pmod{n} \), and dividing by \( a \) in \( \mathbb{Z}_n \) is the same as multiplying by its reciprocal \( x \).

Let \( p \) be prime, and let \( A \) be a matrix with entries in \( \mathbb{Z}_p \). Modify your program from Lab 17, replacing division by an integer \( a \) with multiplication by the inverse of \( a \pmod{p} \), so that it gives the inverse of \( A \) as a matrix with entries in \( \mathbb{Z}_p \). Your output should be a matrix with integer entries, which can be interpreted as elements of \( \mathbb{Z}_p \).

(2) Write a function which takes a prime \( p \) and a positive integer \( n \) and a list of integers \( x_1, x_2, \ldots, x_n \) and returns the matrix
\[
\begin{bmatrix}
1 & x_1^2 & \cdots & x_1^{n-1} \\
1 & x_2^2 & \cdots & x_2^{n-1} \\
\vdots & \vdots & \ddots & \vdots \\
1 & x_n^2 & \cdots & x_n^{n-1}
\end{bmatrix} \quad (\text{mod } p).
\]
Your matrix should be a list of lists, as we have done in previous assignments.

(3) You should have received a secret message consisting of three numbers \( x, y, p \). This message cannot be decoded unless at least four people work together, but any set of four students from the class should be able to decrypt the message. Here \( p \) is a large prime and \( (x, y) \) is a point on a cubic polynomial, so that \( y \equiv M + a_1x + a_2x^2 + a_3x^3 \pmod{p} \) for some unknown values \( M, a_1, a_2, a_3 \). By combining your secret message with the secret messages of three other students, you should have enough information to solve the system of equations
\[
y_1 \equiv M + s_1x_1 + s_2x_1^2 + s_3x_1^3 \pmod{p},
\]
\[
y_2 \equiv M + s_1x_2 + s_2x_2^2 + s_3x_2^3 \pmod{p},
\]
\[
y_3 \equiv M + s_1x_3 + s_2x_3^2 + s_3x_3^3 \pmod{p},
\]
\[
y_4 \equiv M + s_1x_4 + s_2x_4^2 + s_3x_4^3 \pmod{p},
\]
or (in matrix form)
\[
\begin{bmatrix}
1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\
1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\
1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\
1 & x_4 & x_4^2 & \cdots & x_4^{n-1}
\end{bmatrix}
\begin{bmatrix}
M \\
s_1 \\
s_2 \\
s_3
\end{bmatrix}
\equiv
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix} \quad (\text{mod } p).
\]

Find the number \( M \pmod{p} \) and use your function \text{int2string100} to find the secret message.