Math 495R Homework 21

In this lab you will use the python `numpy` module to perform the Gram-Schmidt process on a collection of vectors to find an orthonormal basis for their span. To import this module use the command

```
>>> from numpy import *
```

Use the numpy array constructor for vectors:

```
>>> A=array([1,2,3])
```

This will allow you to use the built-in method to compute dot products. To compute the dot product of two vectors `A` and `B`, use the syntax `A.dot(B)` or `B.dot(A)`.

(1) Write a function `Projection(A,B)` that takes as input two vectors `A` and `B` of the same length n and computes the projection of `A` onto the subspace of $\mathbb{R}^n$ spanned by `B`. Recall the formula for this projection is

$$\mathrm{Proj}_{\mathbf{B}}(\mathbf{A}) = \frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{B}\cdot\mathbf{B}}\mathbf{B},$$

where $\cdot$ is the usual dot product.

(2) Write a function `GramSchmidt(X)` that takes as input a list `X` of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \dots \mathbf{v_p}\}$ and returns a list of vectors $\{\mathbf{u_1}, \mathbf{u_2}, \dots \mathbf{u_q}\}$ which forms an orthogonal basis for the space spanned by the original vectors. You may assume the original vectors are the same length, however you should not assume the original vectors are linearly independent. If the original vectors are linearly *dependent*, you will compute some $\mathbf{u_i} = \mathbf{0}$. Do not included any $\mathbf{0}$ vectors in your orthogonal basis.

(3) Write a function `Orthonormal(X)` that takes as input a list `X` of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \dots \mathbf{v_p}\}$ and returns a list of vectors $\{\mathbf{u_1}, \mathbf{u_2}, \dots \mathbf{u_q}\}$ which forms an orthonormal basis for the space spanned by the original vectors. Use the `GramSchmidt` function from the previous part to construct an orthogonal basis, and then normalize each element to have norm 1.