Math 495R Homework 13

In this lab we will study Riemann sums for higher dimensions

(1) Write a function `riemann_sum_2D` which takes 7 parameters `f, xMin, xMax, yMin, YMax, N`, and `method` and returns the Riemann sum

$$\sum_{j=1}^{N}\sum_{i=1}^{N} f(x_i^*, y_j^*)\Delta x\Delta y$$

where $\Delta x = (\texttt{xMax} - \texttt{xMin})/N$, $\Delta y = (\texttt{yMax} - \texttt{yMin})/N$ and `method` determines whether we are using the lower left, upper right, or midpoints of the partition. (The options for the method should be `left, right, mid`).

(2) Using your function from problem 1, calculate the Riemann sum for $N = 10$ and $N = 20$ on the following problems
   (a) $f(x, y) = x\sin(xy)$ on the rectangle $[0, \pi] \times [0, \pi]$.
   (b) $f(x, y) = y^2 e^{-x-y}$ on the rectangle $[0, 1] \times [0, 1]$.
   (c) $f(x, y) = x^3y^2 + xy$ on the rectangle $[0, 1] \times [1, 2]$.

(3) Consider $f(x, y) = x\sin(x + y)$ on the rectangle $[0, \pi/6] \times [0, \pi/3]$. First calculate the value of this integral analytically. Then make a plot that show the error of the Riemann integral approximation as N ranges from 1 to 100.

(4) Write a `riemann_sum_3D` which takes 9 parameters `f, xMin, xMax, yMin, YMax, zMin, zMax, N`, and `method` and returns the Riemann sum

$$\sum_{k=1}^{N}\sum_{j=1}^{N}\sum_{i=1}^{N} f(x_i^*, y_j^*)\Delta x\Delta y\Delta z,$$

where $\Delta x = (\texttt{xMax} - \texttt{xMin})/N$, $\Delta y = (\texttt{yMax} - \texttt{yMin})/N$, $\Delta z = (\texttt{zMax} - \texttt{zMin})/N$ and `method` determines whether we are using the lower left, upper right, or midpoint of the partition. (Hint you can copy much of the code you wrote for Problem 1.)

(5) Using your function from problem 4, calculate the Riemann sum for $N = 10$ and $N = 20$ on the following problem

$$f(x, y, z) = xy + z^2 \text{ on the rectangle } [0, 2] \times [0, 1] \times [0, 3].$$