# Math 112 (Calculus I)
## Final Exam
### December 15, 2009

**Instructions:**
- Work on scratch paper will not be graded.
- In the multiple choice and short answer sections, **only** the answer will be graded. There will be no partial credit.
- For questions 10 to 20, partial credit will be given if you show **all** your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, $e^0$, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

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**For Instructor use only.**

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Multiple Choice. Fill in the answer to each problem on your computer-scored answer sheet. Make sure your name, section and instructor are on that sheet.

1. Approximate \( \int_1^5 x^4 \, dx \) using a Left Hand sum with 2 subintervals (n=2).
   (a) 82 (b) 164 (c) 81 (d) 162 (e) 624 (f) 625 (g) None of these

2. Find the area under the function \( f(x) = \sqrt[3]{x} \) from \( x = 1 \) to \( x = 8 \).
   (a) \( \frac{45}{4} \) (b) \( \frac{1}{4} \) (c) 12 (d) 15 (e) \( \frac{1}{12} \) (f) None of these

3. Given the limit statement \( \lim_{x \to 1} (2x - 3) = -1 \) pick the largest \( \delta \) that works with the definition of the limit if \( \epsilon = 0.06 \).
   (a) 0.001 (b) 0.005 (c) 0.01 (d) 0.02 (e) 0.03 (f) No such \( \delta \) exists

4. Which of the following is an inflection point of \( f(x) = \frac{x}{x^2 + 1} \)?
   (a) 1 (b) \( -1 \) (c) 2 (d) \( -2 \) (e) \( \sqrt{2} \) (f) \( -\sqrt{2} \) (g) \( -3 \) (h) \( \sqrt{3} \)

5. Given \( x \ln y - y \ln x = e^2 - 2e \), find \( \frac{dy}{dx} \) at the point \((e^2, e)\).
   (a) 0 (b) \( e \) (c) \( e^2 \) (d) \( \frac{1-e}{e^2} \) (e) \( \frac{1-e}{e^2 - 2e} \) (f) \( e^2 - 2e \) (g) \( \frac{e-1}{e^2} \)

6. Which of the following are \( x \)-values for which \( f(x) = \sin(x) - x \) has a local maximum?
   (a) \(-2\pi\) (b) \(-\pi\) (c) 0 (d) \( \pi \) (e) \( 2\pi \) (f) More than one of these (g) None of these

7. Which of the following functions has a discontinuous first derivative?
   (a) \( \sinh(x) \) (b) \( x^{1/3} \) (c) \( \tan^{-1}(x) \) (d) \( \frac{x}{1+x^2} \) (e) \( \ln(x^2 + 1) \)
   (f) All of the first derivatives of these functions are continuous

8. \( \frac{d}{dx} \int_1^{2x} \sqrt{1 + t^3} \, dt = \)
   a) \( \sqrt{1 + (2x)^3} - \sqrt{2} \) b) \( 2\sqrt{1 + (2x)^3} - \sqrt{2} \) c) \( \sqrt{1 + x^3} - \sqrt{2} \)
   d) \( 2\sqrt{1 + x^3} - \sqrt{2} \) e) \( \sqrt{1 + (2x)^3} \) f) \( 2\sqrt{1 + (2x)^3} \)
   g) \( \sqrt{1 + x^3} \) h) \( 2\sqrt{1 + x^3} \)

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Short Answer: Fill in the blank with the appropriate answer.

9. (11 points)

(a) Simplify $\sin \left( \cos^{-1} \left( \frac{3}{5} \right) \right)$.

(b) $\int x^2 e^{x^3} dx = \ldots$

(c) $\frac{d}{dx} (\ln(x)) = \ldots$

(d) $\frac{d}{dx} (\sin^2(x)) = \ldots$

(e) $\frac{d}{dx} (e^x + x^3) = \ldots$

(f) If $f'(x) = e^x + \sin x + x^2$, then $f(x) = \ldots$

(g) $\lim_{x \to \infty} \tan^{-1}(x) = \ldots$

(h) $\lim_{x \to 2} \frac{x + 2}{x^2 - 4} = \ldots$

(i) $\lim_{x \to 0^+} \sin x - \ln x = \ldots$

(j) $\int_1^2 4 + 5x \, dx = \ldots$

(k) $\frac{d}{dx} (2^x) = \ldots$

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Free response: Write your answer in the space provided.

10. (8 points)
   (a) If \( f(x) = \frac{1}{x} \), use the definition of a derivative to set up a limit to find \( f'(x) \).

   (b) Find \( f'(x) \) by evaluating the limit. (No points will be awarded if differentiation rules are used.)

11. (6 points) Find the dimension of the largest rectangle that can be inscribed between the curve \( y = 4 - x^2 \) and the \( x \)-axis.
12. (4 points) \( \lim_{{x \to 0}} \ln(x) \sin(x) \)

13. (3 points) \( \frac{d}{dx} \left( \ln \left( xe^x - \frac{\sin x}{x} \right) \right) \)

14. (4 points) \( \int \frac{x}{x^2 + 4} \, dx \)
15. (10 points) Give the following information about the function $f(x) = x^4 - 4x^3$: (If no information is available in a particular category, leave it blank or cross it out. Putting information in where none exists will be treated as an incorrect answer).

All $x$-intercepts = ____________

$y$-intercept = ____________

Intervals for which $f(x)$ is increasing: ____________

Intervals for which $f(x)$ is decreasing: ____________

Coordinates of all inflection points: ____________

Intervals for which $f(x)$ is concave up: ____________

Intervals for which $f(x)$ is concave down: ____________

Coordinates of any local maximums: ____________

Coordinates of any local minimums: ____________

Graph the function:
16. (6 points) A certain element has a half life of 20 years. How many years will it take until only 10% of the element remains? (Note: $\ln \left(\frac{1}{2}\right) \approx -0.7$ and $\ln \left(\frac{1}{10}\right) \approx -2.3$. You can either leave your answer in terms of logs or give a numerical answer using these approximations.)

17. (6 points) The equation of the tangent line to the curve $y = \frac{1}{x^2}$ at $(2, \frac{1}{4})$.

18. (6 points) Use linear approximation to estimate $\sqrt{63}$:
19. (6 points) A pump is blowing up a spherical balloon with a pump rate of $10\text{cm}^3/\text{sec}$. How fast is the diameter of the balloon growing when the balloon has a 5cm radius? (Volume of a sphere is given by $\frac{4}{3}\pi r^3$.)

20. (6 points) A particle is moving with the given data. Find the position function of the particle.

\[ a(t) = \sin(t) + 3\cos(t), \quad s(0) = 0, \quad v(0) = 2. \]