Name: ____________________

Student ID: ____________________

Section:(See Bubble Sheet) ____________________

Instructor: ____________________

Math 112 (Calculus I)
Final Exam Form B

Dec 13, 2011, 7:00 - 10:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 21 to 30, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned
MC	40	
21	3	
22	5	
23	5	
24	7	
25	7	
Sub	67	

#	Possible	Earned
26	4	
27	8	
28	9	
29	7	
30	5	
Sub	33	
Total	100	

Part I: Multiple Choice. Enter your answer on the scantron. Work will not be collected or reviewed.

1. Let $f(x) = 3x^5 + 5x^4 + 7$. On which of the following intervals is f increasing?

   a) $(-\infty, \infty)$ b) $(-\infty, -1)$ and $(0, \infty)$ c) $(-1, 0)$

   d) $(-4/3, 0)$ e) $(-1, \infty)$ f) $(-\infty, -4/3)$ and $(0, \infty)$

   g) None of these.

2. Let $G(x)$ be the function

$$G(x) = \int_x^{x^2} t \cos t \, dt.$$

   Find the *derivative* $G'(x)$

   a) $x \cos x$ b) 0 c) 1

   d) $2x^3 \cos x^2 - x \cos x$ e) $\cos x^2 + x^2 \sin x^2 - \cos x - x \sin x$ f) $x^2 \cos x^2$

   g) $\sin x^2 - \sin x$ h) None of these

3. Find $f'(x)$ where $f(x) = \ln(x^2 + 2)$.

   a) $\ln x^2 + 2$ b) $2x \ln(x^2 + 2)$ c) $x \ln(x^2 + 2)$ d) $\dfrac{1}{x^2 + 2}$

   e) $\dfrac{2x}{x^2 + 2}$ f) $\dfrac{1}{2x}$ g) $\ln 2x$ h) $\dfrac{x}{x^2 + 2}$

   i) None of the above.

4. If for all x you know that $2x^2 + x - 2 \leq f(x) \leq 4x^4 + 2x^2 + x - 2$, do you have enough information to find $\lim_{x \to 0} f(x)$? If so, what is $\lim_{x \to 0} f(x)$?

   a) Yes, 2 b) Yes, 0

   c) Yes, -2 d) No, not enough information.

   e) Yes, -1 f) Yes, 1

   g) Yes, but none of the above numbers.

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5. Find the derivative $h'(x)$ of the function $h(x) = \dfrac{3e^x + 2x}{\sin x}$.

a) $\dfrac{(3e^x + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$

b) $\dfrac{3e^x + 2}{\cos x}$

c) $\dfrac{(3e^x + 2)\sin x}{\sin^2 x}$

d) $\dfrac{3xe^{x-1} + 2}{\cos x}$

e) $\dfrac{2}{\sin^2 x}$

f) $\dfrac{(3e^x + 2x)\cos x}{\sin^2 x}$

g) $\dfrac{(3xe^{x-1} + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$

h) None of these.

6. Find $\dfrac{dy}{dx}$ where $xy = \cos y$.

a) $-\dfrac{y}{(x + \sin y)}$

b) $-\sin y$

c) $\dfrac{\cos y}{x}$

d) $-\dfrac{\sin y + y}{x}$

e) $-\dfrac{x \sin y + \cos y}{x^2}$

f) None of the above.

7. Find

$$\lim_{x \to \infty} \frac{5 - 3x^3}{\sqrt{81x^6 - 16}}.$$

a) $-\infty$

b) $\dfrac{1}{3}$

c) Does not exist

d) 3

e) -3

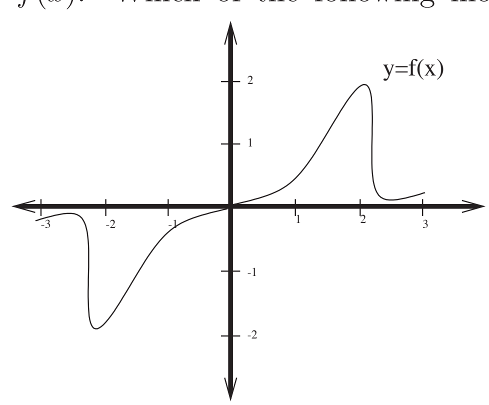
f) 1

g) -1

h) $-\dfrac{1}{3}$

i) 0

8. The following is the graph of a function $y = f(x)$. Which of the following most closely approximates the *definite integral* $\displaystyle\int_{-2}^{2} f(x)\,dx$?



a) 6

b) 2

c) 0

d) -6

e) -2

f) -4

g) 4

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9. Find the derivative $g'(x)$ of the function $g(x) = x^2 \cos x$.

a) $-2x \sin x$ b) $\cos 2x$ c) $2x \cos x - x^2 \sin x$

d) $-\sin 2x$ e) $2x \sin x + x^2 \cos x$ f) $2x \sin x$

g) $-2x^3 \sin x \cos x$ h) None of these.

10. If a function f is defined and twice differentiable on $(-\infty, \infty)$, $f'(2) = 0$, and $f''(2) = 4$, then

a) f is increasing in a neighborhood around $x = 2$.

b) f has a relative minimum at $x = 2$.

c) f has a relative maximum at $x = 2$.

d) f has an inflection point at $x = 2$.

e) f is decreasing in a neighborhood around $x = 2$.

f) we don't have enough information to prove that any of these are true.

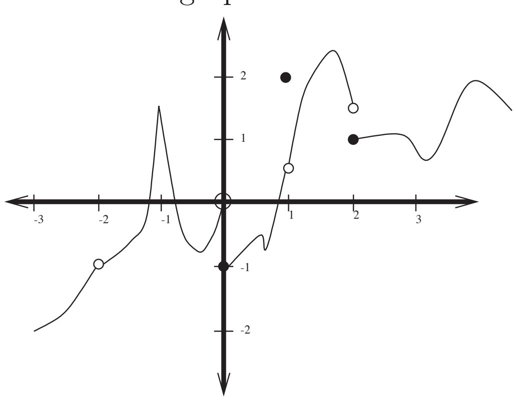
11. Find $\lim_{x \to 2} \left(\dfrac{x^2 - 2x}{x - 2} \right)$.

a) $-\infty$ b) -1 c) -2

d) 1 e) Does not exist f) ∞

g) 2 h) 0

12. Below is the graph of a function. At which, if any, of the following points is it continuous?



a) $x = -2$

b) $x = -1$

c) $x = 2$

d) $x = 1$ and $x = -2$

e) $x = 1$

f) $x = 0$

g) f is not continuous at any of these points.

h) f is continuous at all of these points.

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13. A rectangle is five times as long as it is wide. Its area is increasing at the rate of $9\,\text{cm}^2/s$. Find the rate at which the length is increasing when the width is $3\,\text{cm}$.

a) 45 cm/s b) 1 cm/s c) 3 cm/s d) 3/2 cm/s

e) 3/10 cm/s f) 15 cm/s g) 9 cm/s h) 0 cm/s

i) None of the above.

14. If f is a function that is continuous on the entire real line, with $f(0) = 3$ and $f(1) = 6$, which, if any, of the following theorems guarantees that there is a $c \in (0, 1)$ such that $f(c) = 5$?

a) Extreme Value Theorem

b) Mean Value Theorem

c) There is no theorem that guarantees this, because it is not always true.

d) The Fundamental Theorem of Calculus

e) Intermediate Value Theorem

f) Rolle's Theorem

g) There is a theorem that guarantees this, but it is not in the list.

15. Calculate $\displaystyle\int_0^{\sqrt{\ln 2}} x\, e^{x^2+2}\, dx$.

a) 2 b) e^2 c) $e^2/2$

d) e e) 1 f) 1/2

g) 0 h) $\ln 2$ i) None of these

16. Find $f'(x)$ where $f(x) = (x^3 + 5x + 11)^7$.

a) $(x^3 + 5x + 11)^7$ b) $(3x^2 + 5)$ c) $7(x^3 + 5x + 11)^6$

d) $7(3x^2 + 5)^6$ e) $7(x^3 + 5x + 11)^6(3x^2 + 5)$ f) None of the above.

17. What is the maximum value of $f(x) = 4x^2 - x^4 + 1$ on the interval $[-2, 2]$?

a) $y = 3$ b) $y = 4$ c) $y = 1$

d) $y = 9$ e) $y = 0$ f) $y = 5$

g) $y = 2$ h) $y = 6$ i) None of these.

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18. A culture of *Bacillus Thuringiensis* grows exponentially, and it doubles in size after 1 hour. How long will it take to triple in size?

a) It will never be triple its original size. b) 0 hours c) 1 hour

d) $1\frac{1}{2}$ hours e) $\ln 3$ hours f) $\dfrac{\ln 2}{\ln 3}$ hours

g) $\dfrac{\ln 3}{\ln 2}$ hours h) $\ln 2$ hours i) None of these.

19. Let $h(x) = f(g(x))$, and let $g(2) = 1$, $g'(2) = 2$, $f(1) = 3$, $f'(1) = 5$, $f(2) = 3$, and $f'(2) = 7$. Find $h'(2)$.

a) 35 b) 21 c) 10

d) 15 e) 7 f) 14

g) 28 h) 2 i) 5

j) None of the above.

20. Find $\lim_{x\to 2} \left(\dfrac{|x-2|}{x-2} \right)$.

a) ∞ b) -1 c) 0

d) -2 e) 2 f) Does not exist

g) 1 h) $-\infty$

Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

21. (3 points) Given $\epsilon > 0$, and the statement

$$\lim_{x\to -1}(-3x+1) = 4,$$

find the largest $\delta > 0$ so that the following statement is true:
"If $0 < |x+1| < \delta$, then $|-3x+1-4| < \epsilon$."
Do NOT prove the statement.

22. (5 points) Use the definition of the derivative to show:

$$\text{If } f(x) = \frac{1}{x+2}, \text{ then } f'(x) = -\frac{1}{(x+2)^2}.$$

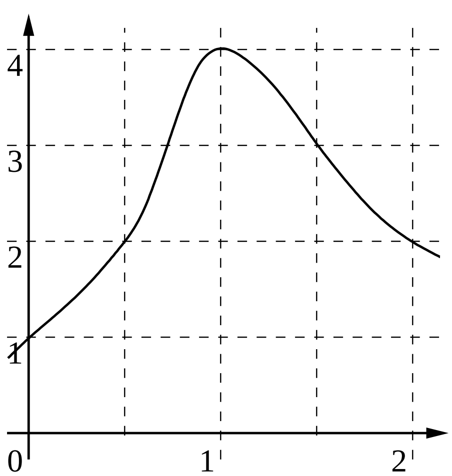
No credit will be given if a method besides the definition of the derivative is used.

23. (5 points) Use linear approximation to estimate $\sqrt{26}$.

24. (7 points) Prove that $f(x) = -2x + \cos x$ has at most one root.

25. (7 points) Find the limit. $\lim_{x\to\infty} \left(\frac{1}{x^4}\right)^{1/x}$.

26. (4 points) Let $f(x)$ be the function whose graph is shown below. Use **left** hand sums with four rectangles to estimate $\int_0^2 f(x)dx$.



27. (8 points) Suppose the area of a right triangle is 32 cm^2. Find the smallest possible length of its hypoteneuse.

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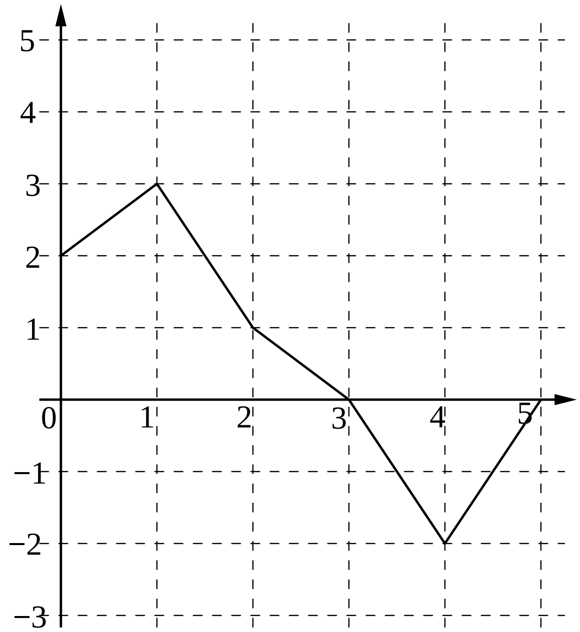
28. (9 points) The graph of a function $f(x)$ is shown. Let $g(x) = \int_0^x f(t)dt$ for $0 \leq x \leq 5$.



(a) Evaluate $g(0)$, $g(3)$, and $g(5)$.

(b) Where is $g(x)$ increasing on $[0, 5]$? Decreasing?

(c) Where is $g(x)$ concave up on $[0, 5]$? Concave down?

(f) Sketch the graph of $g(x)$ on the same axes of $f(x)$, labelling all local maxima and minima, and all points of inflection.

29. (7 points) A pendulum swings with velocity $v(t) = \sin t$.

(a) Find the displacement of the pendulum for $0 \leq t \leq \dfrac{5\pi}{2}$.

(b) Find the total distance the pendulum travels over $0 \leq t \leq \dfrac{5\pi}{2}$.

30. (5 points) Evaluate the integrals.

(a) $\displaystyle\int \frac{1 + (\ln x)^2}{x} dx$

(b) $\displaystyle\int \frac{-dx}{\sqrt{1 - x^2}}$

END OF EXAM