Instructions:

• The time limit is 3 hours.

• Problem 1 consists of fill in the blank questions, each worth 1 point.

• Problems 2 through 8 are multiple choice questions, each worth 4 points.
  Their answers **MUST** be entered on the grid on page 2.

• Work on scratch paper will not be graded. Do not show your work for problem 1 through 8.

• Write solutions to problems 9 through 18 on the exam paper in the space provided.
  Problems 9–17 are worth 6 points each.
  Problem 18 is worth 9 points (3 points per part).
  You must show your work to receive full credit.

• Please write neatly, and simplify your answers.

• Notes, books, and calculators are not allowed.

• Expressions such as \( \ln(1), e^0, \sin(\pi/2) \), etc. must be simplified for full credit.

For administrative use only:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/9</td>
<td></td>
</tr>
<tr>
<td>M.C.</td>
<td>/28</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>/6</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>/9</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>/100</td>
<td></td>
</tr>
</tbody>
</table>
1. (a) The limit \( \lim_{x \to \infty} \frac{2x^2 + 3x}{1 - x^2} = \)_______________________

(b) If \( f(x) = x^2 \), and \( x_0 = 1 \), then Newton’s method for solving \( f(x) = 0 \) gives us \( x_1 = \)_______________________

(c) ( T/F ) If \( f''(x) \) exists on \([a, b]\), then \( f(x) \) is continuous on \([a, b]\)

(d) The mean value theorem states that if \( f \) is differentiable on \([a, b]\), then there is a \( c \) in \((a, b)\) with

\[ f'(c) = \]_______________________

(e) The limit \( \lim_{x \to 3^+} \frac{|x - 3|}{x - 3} = \)_______________________

(f) The average value of a function \( f \) over an interval \([a, b]\) is given by

\[ \frac{1}{b-a} \int_a^b f(x) \, dx = \]_______________________

(g) If \( \int_2^4 f(x) \, dx = 2 \), \( \int_0^4 f(x) \, dx = 6 \), \( \int_0^2 g(x) \, dx = 5 \), then

\[ \int_0^2 f(x) + 3g(x) \, dx = \)_______________________

(h) ( T/F ) If \( f'(x) \) exists on \([a, b]\), then \( f(x) \) is integrable on \([a, b]\)

(i) The integral \( \int \frac{dx}{1 + x^2} = \)_______________________
Problems 2 through 8 are multiple choice. Each multiple choice problem is worth 4 points. In the grid below fill in the square corresponding to each correct answer.

2. \( \lim_{x \to 0} \frac{2x}{\sin 5x} = \)
   A. 0 B. \( \frac{2}{5} \) C. 1 D. \( \frac{5}{2} \)
   E. 2 F. \( -\infty \) G. \( \infty \) H. Limit does not exist.

3. Given the limit statement \( \lim_{x \to 5} (-3x + 17) = 2 \), pick the largest \( \delta \) that works with the definition of the limit if \( \epsilon = 0.06 \).
   A. 0.001 B. 0.005 C. 0.01 D. 0.02
   E. 0.03 F. 0.06 G. 0 H. No such \( \delta \).

4. If \( f(x) = 6x^2 \), \( g(-1) = -2 \), \( g'(-1) = 3 \), find \( \frac{d}{dx} (f(g(x))) \) at \( x = -1 \).
   A. 0 B. 1 C. -12 D. -24
   E. -36 F. -48 G. -72 H. None of the above.

5. Which of the following is the maximum value of \( f(x) = 2x^3 - 3x^2 - 36x + 4 \) over \([-3, 2]\)?
   A. 4 B. -2 C. 3 D. 80
   E. 48 F. 64 G. -16 H. None of the above.

6. Given \( x^2 \ln y + y \ln(x^2) = 2e \), find \( \frac{dy}{dx} \) at the point \((\sqrt{e}, e)\).
   A. -1 B. \( e \) C. -2e D. \( \frac{\sqrt{e}}{2\sqrt{e} - 2} \)
   E. -2\( \sqrt{e} \) F. -\( \sqrt{2e} \) G. 1 H. \( \frac{2\sqrt{e} + 2}{\sqrt{e} - 2} \)
7. Which of the following is a Riemann sum for \( \int_0^1 \sinh^{-1} x \, dx \) as \( n \to \infty \)?

A. \( \sum_{j=0}^{n-1} \sinh^{-1} \left( \frac{1}{n} \right) \cdot \frac{j}{n} \)
B. \( \sum_{j=0}^{n-1} \sinh^{-1} \left( \frac{j+1}{n} \right) \cdot \frac{1}{n} \)
C. \( \sum_{j=0}^{n-1} \sinh^{-1} \left( \frac{j}{n} \right) \cdot \frac{j}{n} \)
D. \( \sum_{j=0}^{n-1} \sinh^{-1} \left( \frac{2j+1}{2n} \right) \cdot \frac{1}{n} \)
E. (A) and (C)
F. (B) and (D)
G. All of the above
H. None of the above

8. Evaluate \( \int_0^{1/2} 8(1 - 4x)^3 \, dx \)

A. \(-1\)  B. \(1\)  C. \(4\)  D. \(-4\)
E. \(0\)  F. \(3\)  G. \(-5\)  H. None of the above.

The answers to the multiple choice MUST be entered on the grid on page 2. Otherwise, you will not receive credit.
For problems 9 - 18, write your answers in the space provided. Neatly show your work for full credit.

9. (a) State the conditions for \( f(x) \) defined over \([0, 2]\) to be continuous at \( x = 1 \).

(b) At which points does the function

\[
f(x) = \frac{\sqrt{x + 4}}{(x + 2)(x - 3)}
\]

fail to be continuous? At which points, if any are the discontinuities removable? not removable? Give reasons for your answers.
10. Differentiate the following:

(a) \( f(x) = x^\pi + \sec(\tan x) \)

(b) \( g(x) = \left(\frac{x^2 + 1}{x^2 - 1}\right)^{1/2} \)

11. Find the equation for the tangent line to \( f(x) = e^x \cos(x) \) at the point \((\pi, -e^\pi)\).
12. If $f(x) = \frac{1}{x - 1}$, find $f'(2)$ using the definition of the derivative. (No point will be awarded if differentiation rules are used.)

13. What are the dimensions of the rectangle of largest area that fits in a right triangle with side lengths 3 in, 4 in and 5 in?
14. The area of a square is increasing at $4 \text{ in}^2/\text{s}$. How fast is the length of the diagonal increasing at the moment that the side of the square is 6 in?

15. Given that for all $x > -4$, the function $f(x)$ is defined, continuous and satisfies the bounds

$$\frac{2}{1 + e^{-1/x^2}} \leq f(x) \leq 2 + \frac{x}{4 - \sqrt{x} + 4}$$

Determine the value $f(0)$. State any theorem you used to find your answer.
16. Let \( A(x) = \int_{x+1}^{\sqrt{x}} \sin t^2 \, dt \). Find \( \frac{dA}{dx} \). State any theorem you used to find your answer.

17. If \( f(x) = \frac{x}{x^2 + 1} \), find all intervals of monotonicity, all intervals of concavity, all inflection points, all relative extrema and all global extrema if possible.
18. Find the following integrals

(a) \( \int_{e^2} e^3 x^{-1} \, dx \)

(b) \( \int \frac{\cos^4(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} \, dx \)

(c) \( \int_0^1 30x \sqrt{1-x} \, dx \)

—End—