Name________________________________

Student Number______________________

Section Number______________________

Instructor__________________________

Math 112 – Winter 2007

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of fill in the blank questions, each worth 1 point.
- Problems 2 through 8 are multiple choice questions, each worth 4 points.
  Their answers **MUST** be entered on the grid on page 2
- Work on scratch paper will not be graded. Do not show your work for problem 1 through 8.
- Write solutions to problems 9 through 18 on the exam paper in the space provided.
  Problems 9–17 are worth 6 points each.
  Problem 18 is worth 9 points (3 points per part).
  You must show your work to receive full credit.
- Please write neatly, and simplify your answers.
- Notes, books, and calculators are not allowed.
- Expressions such as ln(1), e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

1	/9
M.C.	/28
9	/6
10	/6
11	/6
12	/6

13	/6
14	/6
15	/6
16	/6
17	/6
18	/9
Total	/100

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Part I: Fill in the blank or circle T/F

1. (a) The limit $\lim_{x\to\infty} \frac{2x^2+3x}{1-x^2} =$ ________________________________

(b) If $f(x) = x^2$, and $x_0 = 1$, then Newton's method for solving $f(x) = 0$ gives us $x_1 =$ ____________

(c) (T/F) If $f''(x)$ exists on $[a,b]$, then $f(x)$ is continuous on $[a,b]$

(d) The mean value theorem states that if f is differentiable on $[a,b]$, then there is a c in (a,b) with

$f'(c) =$ ________________________________

(e) The limit $\lim_{x\to 3+} \frac{|x-3|}{x-3} =$ ________________________

(f) The average value of a function f over an interval $[a,b]$ is given by

____

(g) If $\int_2^4 f(x)\,dx = 2$, $\int_0^4 f(x)\,dx = 6$, $\int_0^2 g(x)\,dx = 5$, then

$$\int_0^2 f(x) + 3g(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(h) (T/F) If $f'(x)$ exists on $[a,b]$, then $f(x)$ is integrable on $[a,b]$

(i) The integral $\int \frac{dx}{1+x^2} =$ ________________________

Part II: Multiple Choice

Problems 2 through 8 are multiple choice. Each multiple choice problem is worth 4 points. In the grid below fill in the square corresponding to each correct answer.

2	A	B	C	D	E	F	G	H	I	J
3	A	B	C	D	E	F	G	H	I	J
4	A	B	C	D	E	F	G	H	I	J
5	A	B	C	D	E	F	G	H	I	J
6	A	B	C	D	E	F	G	H	I	J
7	A	B	C	D	E	F	G	H	I	J
8	A	B	C	D	E	F	G	H	I	J

2. $\lim_{x \to 0} \dfrac{2x}{\sin 5x} =$

   A. 0 B. $\frac{2}{5}$ C. 1 D. $\frac{5}{2}$
   E. 2 F. $-\infty$ G. ∞ H. Limit does not exist.

3. Given the limit statement $\lim_{x \to 5}(-3x+17) = 2$, pick the largest δ that works with the definition of the limit if $\epsilon = 0.06$.

   A. 0.001 B. 0.005 C. 0.01 D. 0.02
   E. 0.03 F. 0.06 G. 0 H. No such δ.

4. If $f(x) = 6x^2,\ g(-1) = -2,\ g'(-1) = 3$, find $\dfrac{d}{dx}\left(f(g(x))\right)$ at $x = -1$.

   A. 0 B. 1 C. -12 D. -24
   E. -36 F. -48 G. -72 H. None of the above.

5. Which of the following is the maximum value of $f(x) = 2x^3 - 3x^2 - 36x + 4$ over $[-3, 2]$?

   A. 4 B. -2 C. 3 D. 80
   E. 48 F. 64 G. -16 H. None of the above.

6. Given $x^2 \ln y + y \ln(x^2) = 2e$, find $\dfrac{dy}{dx}$ at the point $(\sqrt{e}, e)$.

   A. -1 B. e C. $-2e$ D. $\dfrac{\sqrt{e}}{2\sqrt{e}-2}$
   E. $-2\sqrt{e}$ F. $-\sqrt{2e}$ G. 1 H. $\dfrac{2\sqrt{e}+2}{\sqrt{e}-2}$

7. Which of the following is a Riemann sum for $\int_0^1 \sinh^{-1} x\,dx$ as $n \to \infty$?

A. $\displaystyle\sum_{j=0}^{n-1} \sinh^{-1}\left(\frac{1}{n}\right) \cdot \frac{j}{n}$ B. $\displaystyle\sum_{j=0}^{n-1} \sinh^{-1}\left(\frac{j+1}{n}\right) \cdot \frac{1}{n}$ C. $\displaystyle\sum_{j=0}^{n-1} \sinh^{-1}\left(\frac{j}{n}\right) \cdot \frac{j}{n}$

D. $\displaystyle\sum_{j=0}^{n-1} \sinh^{-1}\left(\frac{2j+1}{2n}\right) \cdot \frac{1}{n}$ E. *(A) and (C)* F. *(B) and (D)*

G. All of the above H. None of the above

8. Evaluate $\displaystyle\int_0^{1/2} 8(1-4x)^3\,dx$

A. -1 B. 1 C. 4 D. -4
E. 0 F. 3 G. -5 H. None of the above.

The answers to the multiple choice MUST be entered on the grid on page 2. Otherwise, you will not receive credit.

PART III: WRITTEN SOLUTIONS

For problems 9 - 18, write your answers in the space provided. Neatly show your work for full credit.

9. (a) State the conditions for $f(x)$ defined over $[0, 2]$ to be continuous at $x = 1$.

   (b) At which points does the function

$$f(x) = \frac{\sqrt{x+4}}{(x+2)(x-3)}$$

   fail to be continuous? At which points, if any are the discontinuities removable? not removable? Give reasons for your answers.

10. Differentiate the following:

(a) $f(x) = x^{\pi} + \sec(\tan x)$

(b) $g(x) = \left(\dfrac{x^2+1}{x^2-1}\right)^{1/2}$

11. Find the equation for the tangent line to $f(x) = e^x \cos(x)$ at the point $(\pi, -e^{\pi})$.

12. If $f(x) = \dfrac{1}{x-1}$, find $f'(2)$ using the definition of the derivative. (*No point will be awarded if differentiation rules are used.*)

13. What are the dimensions of the rectangle of largest area that fits in a right triangle with side lengths 3 in, 4 in and 5 in?

14. The area of a square is increasing at 4 in^2/s. How fast is the length of the diagonal increasing at the moment that the side of the square is 6 in?

15. Given that for all $x > -4$, the function $f(x)$ is defined, continuous and satisfies the bounds

$$\frac{2}{1+e^{-1/x^2}} \leq f(x) \leq 2 + \frac{x}{4-\sqrt{x+4}}$$

Determine the value $f(0)$. State any theorem you used to find your answer.

16. Let $A(x) = \int_{x+1}^{\sqrt{x}} \sin t^2 \, dt$. Find $\dfrac{dA}{dx}$. State any theorem you used to find your answer.

17. If
$$f(x) = \frac{x}{x^2+1},$$
find all intervals of monotonicity, all intervals of concavity, all inflection points, all relative extrema and all global extrema if possible.

18. Find the following integrals

(a) $\displaystyle\int_{e^2}^{e^3} 2x^{-1}\,dx$

(b) $\displaystyle\int \frac{\cos^4(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx$

(c) $\displaystyle\int_0^1 30x\sqrt{1-x}\,dx$

—*End*—