Name:____________________

Student ID:____________________

Section:____________________

Instructor:____________________

Math 112 (Calculus I)
Final Exam Form A

April 18, 7:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 10 to 17, show all your work in the space provided.. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is alloted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned
MC	24	
9	11	
10	7	
11	7	
12	7	
13	7	
14a	3	
Sub	66	

#	Possible	Earned
14b	3	
14c	3	
15a	7	
15b	7	
16	7	
17	7	
Sub	34	
Total	100	

Multiple Choice. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. $\dfrac{d}{dx}\displaystyle\int_3^{4x} \frac{t^3}{\sqrt{1+t^5}}\,dt =$

   a) $\dfrac{256x^3}{\sqrt{1+1024x^5}}$ b) $\dfrac{256x^3}{\sqrt{1+1024x^5}} - \dfrac{27}{\sqrt{244}}$ c) $\dfrac{64x^3}{\sqrt{1+1024x^5}}$

   d) $\dfrac{4x^3}{\sqrt{1+x^5}}$ e) $\dfrac{4x^3}{\sqrt{1+x^5}} - \dfrac{27}{\sqrt{244}}$ f) $\dfrac{x^3}{\sqrt{1+x^5}}$

2. $\displaystyle\int_{\sqrt{5}}^{2\sqrt{3}} \frac{z}{(4+z^2)^{3/2}}\,dz =$

   a) $-\dfrac{1}{7}$ b) $-\dfrac{1}{12}$ c) Does not exist.

   d) $\dfrac{1}{12}$ e) $\dfrac{1}{7}$ f) $\dfrac{1}{4}$

   g) None of the above.

3. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs. (All answers are in square cm.)

   a) 1 b) $\dfrac{1}{3}$ c) 2

   d) 3 e) $\dfrac{9}{2}$ f) There is no largest rectangle.

   g) None of the above.

4. $\displaystyle\lim_{x\to 4} \frac{4-x}{|4-x|} =$

   a) 0 b) 1 c) -1

   d) ∞ e) $-\infty$ f) Does not exist.

   g) None of the above.

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5. We say $\lim_{x \to a} f(x) = L$ if

a) For every $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

b) There exists $\epsilon > 0$ such that for every $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

c) For some $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

d) For every $\epsilon > 0$ and for every $\delta > 0$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

e) For every $\delta > 0$ there exists a $\epsilon > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

f) For some $\epsilon > 0$ and every $\delta > 0$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

g) None of the above.

6. If $f'(x) = e^{x^2}$ and $g(x) = f(\sqrt{x})$, then $g'(x) =$

a) $\dfrac{e^{x^2}}{2x}$ b) $\dfrac{e^{x^2}}{2\sqrt{x}}$ c) $\dfrac{e^{x}}{2\sqrt{x}}$

d) $\left(\dfrac{2x - 1}{4x\sqrt{x}}\right) e^x$ e) $2xe^{\sqrt{x}}$ f) $2\sqrt{x}e^x$

g) None of the above.

7. $\dfrac{d}{dx}\left(x^{\sin x}\right) =$

a) $\sin(x)x^{\sin x - 1}$ b) $\cos(x)x^{\sin x - 1}$ c) $x^{\sin x} \cos x \ln x$

d) $\dfrac{\sin x}{x} x^{\sin x} \cos x$ e) $x^{\sin x}\left(\dfrac{\sin x}{x} + \cos(x) \ln x\right)$ f) $x^{\sin x} \sin x \cos x$

g) None of the above.

8. If \$5000 is borrowed at 5% interest compounded continuously, then the amount due at the end of ten years is

a) $\$5000(1.05)^{10}$ b) $\$5000\sqrt{e}$ c) \$7500

d) $\$5000e$ e) $\$5000e^{1.5}$ f) $\$5000e^{1.05}$

g) None of the above.

Short Answer Fill in the blank with the appropriate answer.

9. (11 points)

a) $\lim_{x \to \pi^-} \ln(\sin x) =$ ____________________

b) What kind of discontinuity exists at $x = -1$ for the function $f(x) = \dfrac{x+1}{x^2-1}$? ____________________

c) $\dfrac{d}{dx}(a^3 + \cos^3 x) =$ ____________________

d) $\dfrac{d^2}{dx^2}(e^{x^2}) =$ ____________________

e) If $f'(x) \geq 2$ for all $x \in [0, 2]$, what theorem tells us that $f(2) - f(0) \geq 4$? ____________________

f) $\dfrac{d}{dx}(\tan^{-1}(x^2)) =$ ____________________

g) $\lim_{x \to 0^+} \dfrac{\ln(1+x)}{x} =$ ____________________

h) $\int (\sqrt{x} + \frac{1}{x})\, dx =$ ____________________

i) $\int_3^4 (1 + 3x)\, dx =$ ____________________

j) $\int_2^5 (2x-1)^2\, dx =$ ____________________

k) Set up a limit to find the derivative of $g(x) = \dfrac{1}{x^2+1}$. ____________________

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Free Response. For problems 10 - 17, write your answers in the space provided. Use the back of the page if needed, indicating that fact. Neatly show all work.

10. (7 points) Prove $\lim_{x\to 2} 4x - 3 = 5$ using the $\epsilon - \delta$ definition of the limit.

11. (7 points) Estimate $\sqrt[3]{8.012}$ by linear approximation.

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12. (7 points) Find an equation to the tangent line to $4x^3 + 2xy + y^3 = 1$ at $(1, -1)$.

13. (7 points) If a ball is thrown vertically upwards from the top edge of a 90 foot building with an initial velocity of 80 feet per second, it's height above the ground (after t seconds) is given by

$$h(t) = 90 + 80t - 16t^2.$$

What is its maximum height above the ground?

14. Let $f(x) = 2x^3 + 3x^2 - 12x$.

(a) (3 points) Find the interval(s) on which $f(x)$ is increasing or decreasing. Label them appropriately.

(b) (3 points) Find the local maxima and minima of $f(x)$

(c) (3 points) Find the intervals of concavity and the inflection points of $f(x)$.

15. Evaluate the following limits:

(a) (7 points) $\lim_{x \to \infty} (\sqrt{x^2 + x} - x)$

(b) (7 points) $\lim_{x \to \infty} x^{1/x}$.

16. (7 points) Find $\int \frac{1 + x}{x + x^2}\, dx$.

17. (7 points) A dog owner has 1000 feet of fencing and wishes to make 4 dog runs side by side. (a dog run is a fenced rectangular area the dog can pace in). What dimensions will give the largest area? (Note: two dog runs that sit side by side share a common side.)

END OF EXAM