Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 112 (Calculus I)
Final Exam Form A

April 17, 2009 at 7:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 9 to 17, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is alloted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned
MC	24	
9a	4	
9b	4	
10	7	
11	7	
12	7	
13a	6	
Sub	59	

#	Possible	Earned
13b	6	
14	6	
15	6	
16	8	
17a	5	
17b	5	
17c	5	
Sub	41	
Total	100	

Multiple Choice. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. Find the absolute *minimum* value for $f(x)$ on the interval $[-4, 3]$ when $f(x)$ is given by

$$f(x) = \frac{x^2 - 4}{x^2 + 4}.$$

a) -1 b) $5/13$ c) $4/5$

d) -2 e) 2 f) 1

2. For what value of c is the function $g(x)$ below continuous?

$$g(x) = \begin{cases} \dfrac{cx^2 - 4c}{x - 2} & \text{if } x < 2 \\ cx + 1 & \text{if } x \geq 2 \end{cases}$$

a) $c = 1$ b) $c = 2$ c) $c = 1/2$

d) $c = -2$ e) $c = 0$ f) None of the above

3. Evaluate:

$$\frac{d}{dx} \int_3^{x^2} \ln(t - 1)\, dt$$

a) $2x \ln(x^2 - 1)$ b) $\dfrac{1}{x^2 - 1} - \dfrac{1}{8}$ c) $\ln(x^2 - 1) - \ln(8)$

d) $2x \ln(x^2 - 1) - 2x \ln(8)$ e) $2x \ln((x^2 - 1)^4 (x^2 - 1))$ f) $\dfrac{2x}{x^2 - 1} + \dfrac{x}{4}$

4. Use linear approximation to estimate $\sinh(0.1)$.

a) 0.2 b) 0.01 c) -0.2

d) 1.1 e) 0.9 f) 0.1

5. A function $s(t)$ is given by

$$s(t) = t^{1/3} + t^{2/3}.$$

Find $s''(1)$.

a) 1 b) $-1/3$ c) 0

d) $-4/9$ e) $1/3$ f) -1

6. Find all vertical asymptotes.

$$y = \frac{3(x-2)(x+1)\ln(x+5)}{(x+4)(x-2)}$$

i. $x = -4$ ii. $x = 2$ iii. $x = -5$ iv. $x = 1$

a) i. only b) i. and ii. c) i., ii., and iii.

d) i., ii., iii., and iv. e) i. and iii. only f) ii. and iii. only

7. For the definition of the limit

$$\lim_{x \to 2} 3x = 6,$$

find the largest value of δ that corresponds to $\epsilon = 0.06$.

a) 0.06 b) 0.02 c) 0.2

d) 0.1 e) 0.03 f) 0.01

8. A certain isotope has a half-life of 40 years. Suppose we have a 100 mg sample. How much remains after 60 years?

a) $100/\sqrt[3]{4}$ mg b) 50 mg c) $100e^{3/2}$ mg

d) $25\sqrt{2}$ mg e) $200/3$ mg f) $100\ln(1/2)$ mg

Short Answer. Fill in the blank with the appropriate answer.

9. A rectangular box with a square base is to hold 4000 cubic centimeters. Material for the sides costs 2 cents per square centimeter, while material for the top and bottom costs 1 cent per square centimeter.

   (a) (4 points) If the base has side x, express the cost C of the box as a function of x.

   (b) (4 points) Find the dimensions of the most economical box.

10. (7 points) Find the equation of the tangent line to the curve given implicitly by

$$e^{xy} = x^2 + y^2$$

at the point $(0, 1)$.

11. (7 points) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 2 m/s, at what rate is the angle between the rope and the horizontal increasing when that angle is $\pi/6$?

12. (7 points) Find the derivative of $(\arctan(x))^{3x}$. *Don't simplify your answer.*

13. Evaluate the following limits.

(a) (6 points) $\lim_{x\to\infty} 3x^2\, e^{-4x}$

(b) (6 points) $\lim_{h\to 0} \dfrac{a^{1+h} - a}{h}$

14. (6 points) Write $\lim_{n\to\infty} \sum_{i=1}^{n} e^{i/n} \left(\dfrac{1}{n}\right)$ as a definite integral and calculate it.

15. (6 points) Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$.

16. (8 points) Show $\ln(x) + x - 2 = 0$ has exactly one real root in the interval $(1, e)$.

17. Evaluate the integrals.

(a) (5 points) $\displaystyle\int_0^1 v^2 \cos(v^3)\, dv$

(b) (5 points) $\displaystyle\int \frac{\sec\theta \tan\theta}{1+\sec\theta}\, d\theta$

(c) (5 points) $\displaystyle\int_0^3 |x^2 - 4|\, dx$

END OF EXAM