Math 112 (Calculus I)
Final Exam Form B KEY

Part I: Multiple Choice. Enter your answer on the scantron. Work will not be collected or reviewed.

1. If a function f is defined and twice differentiable on $(-\infty, \infty)$, $f'(2) = 0$, and $f''(2) = 4$, then

   a) f has a relative minimum at $x = 2$.

   b) f is increasing in a neighborhood around $x = 2$.

   c) f has a relative maximum at $x = 2$.

   d) f has an inflection point at $x = 2$.

   e) f is decreasing in a neighborhood around $x = 2$.

   f) we don't have enough information to prove that any of these are true.

   Solution: a)

2. If $\int_1^6 f(x)\,dx = 8$ and $\int_4^6 f(x)\,dx = 12$, find $\int_1^4 f(x)\,dx$.

   a) 4 b) -3 c) 3 d) 2

   e) -4 f) 20 g) -2

   Solution: e)

3. Find $\lim\limits_{x\to\infty} \dfrac{5 - 3x^3}{\sqrt{81x^6 - 16}}$.

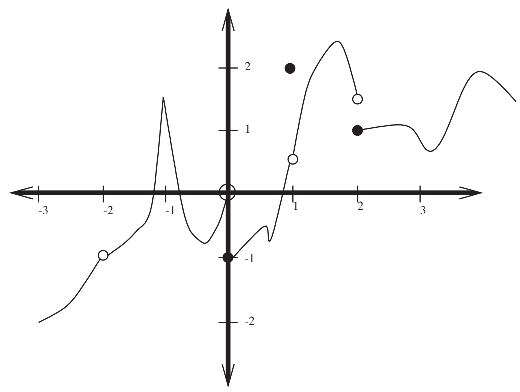
   a) $-\dfrac{1}{3}$ b) 1 c) Does not exist d) -1

   e) -3 f) $-\infty$ g) $\dfrac{1}{3}$ h) 3

   i) 0

   Solution: a)

4. Below is the graph of a function. At which, if any, of the following points is it continuous?



a) $x = 2$ b) $x = 1$ and $x = -2$ c) $x = 1$

d) $x = 0$ e) $x = -2$ f) $x = -1$

g) f is continuous at all of these points. h) f is not continuous at any of these points.

Solution: f)

5. Find $\lim\limits_{x\to 2}\left(\dfrac{x^2-2x}{x-2}\right)$.

a) 0 b) 2 c) Does not exist d) -2

e) ∞ f) 1 g) $-\infty$ h) -1

Solution: b)

6. Find $f'(x)$ where $f(x) = \ln(x^2+2)$.

a) $2x\ln(x^2+2)$ b) $\dfrac{x}{x^2+2}$ c) $\dfrac{2x}{x^2+2}$ d) $\dfrac{1}{2x}$

e) $\ln x^2 + 2$ f) $\ln 2x$ g) $\dfrac{1}{x^2+2}$ h) $x\ln(x^2+2)$

i) None of the above.

Solution: c)

7. Find an antiderivative of $f(x) = 3x^2 + \dfrac{2}{x^2}$.

a) $x^3 + \dfrac{1}{x}$ b) $x^3 - \dfrac{4}{x^3}$ c) $x^3 + \dfrac{4}{x^3}$

d) $x^2 + \dfrac{2}{x^2}$ e) $x^3 - \dfrac{2}{x}$ f) $x^3 + \dfrac{2}{x}$

Solution: e)

8. Let $G(x)$ be the function $G(x) = \displaystyle\int_x^{x^2} t\cos t\,dt$. Find the *derivative* $G'(x)$.

a) $\sin x^2 - \sin x$ b) $\cos x^2 + x^2\sin x^2 - \cos x - x\sin x$

c) 0 d) 1

e) $x\cos x$ f) $x^2\cos x^2$

g) $2x^3\cos x^2 - x\cos x$ h) None of these

Solution: g)

9. Find $\dfrac{dy}{dx}$ where $xy = \cos y$.

a) $-\dfrac{y}{x + \sin y}$

b) $\dfrac{\cos y}{x}$

c) $-\dfrac{x \sin y + \cos y}{x^2}$

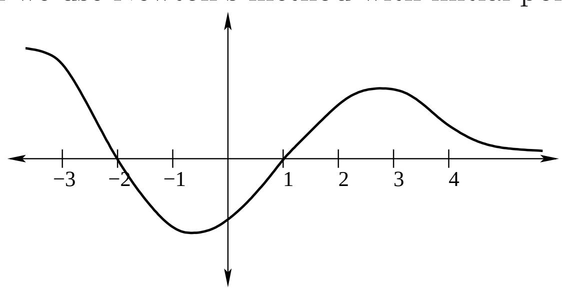
d) None of the above.

e) $-\sin y$

f) $-\dfrac{\sin y + y}{x}$

Solution: a)

10. For the graph shown, if we use Newton's method with initial point $x_1 = 0$, what will happen?



a) We obtain a sequence of points converging to the root at $x = -2$.

b) We obtain a sequence of points diverging to ∞.

c) Newton's method will fail immediately.

d) We obtain a sequence of points diverging to $-\infty$.

e) We obtain a sequence of points converging to the root at $x = 1$.

f) None of the above.

Solution: e)

11. What is the maximum value of $f(x) = 4x^2 - x^4 + 1$ on the interval $[-2, 2]$?

a) $y = 5$

b) $y = 6$

c) $y = 0$

d) $y = 9$

e) $y = 3$

f) $y = 4$

g) $y = 2$

h) $y = 1$

i) None of these.

Solution: a)

12. Water leaks out of a tank at the rate of $r(t) = 200 - 8t$ liters per minute, where $0 \leq t \leq 25$. Find the amount of water that leaks out in the first five minutes.

a) 2500 L

b) 4500 L

c) 900 L

d) -8 L

e) 160 L

f) 1100 L

g) 420 L

h) 860 L

Solution: c)

13. Evaluate $\int \frac{e^t}{(1-e^t)^2}\,dt$.

a) $-\frac{1}{(1-e^t)^3}+C$ b) $\frac{e^t}{(1-e^t)}-\frac{2e^{2t}}{(1-e^t)^3}+C$ c) $-\frac{1}{(1-e^t)^2}+C$

d) $\frac{1}{(1-e^t)}+C$ e) $-\frac{1}{1-e^t}+C$ f) $e^t\ln(1-e^t)^2+C$

g) $\frac{1}{(1-e^t)^3}+C$

Solution: d)

14. Let $k(x)=\sqrt{x-1}$. For what value of c does $k(x)$ satisfy the Mean Value theorem on the interval $[1,5]$? (In other words, what value of c satisfies $k'(c)=\frac{k(5)-k(1)}{5-1}$)?

a) 4 b) 6 c) 2 d) 3

e) 1 f) 5

Solution: c)

15. Find the derivative $g'(x)$ of the function $g(x)=x^2\cos x$.

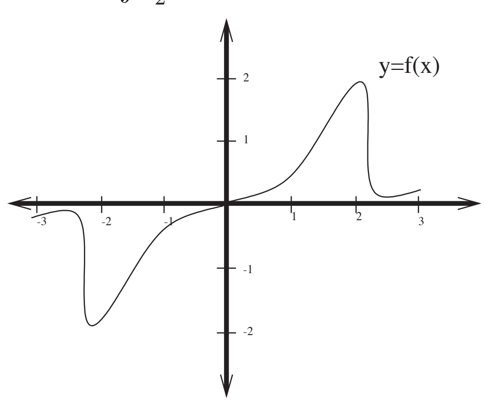
a) $-2x^3\sin x\cos x$ b) $2x\sin x$ c) $2x\cos x-x^2\sin x$

d) $-\sin 2x$ e) $-2x\sin x$ f) $\cos 2x$

g) $2x\sin x+x^2\cos x$ h) None of these.

Solution: c)

16. The following is the graph of a function $y=f(x)$. Which of the following most closely approximates the *definite integral* $\int_{-2}^{2} f(x)\,dx$?



a) 2 b) -6 c) -2 d) 6

e) -4 f) 4 g) 0

Solution: g)

17. Let $h(x) = f(g(x))$, and let $g(2) = 1$, $g'(2) = 2$, $f(1) = 3$, $f'(1) = 5$, $f(2) = 3$, and $f'(2) = 7$. Find $h'(2)$.

a) 2 b) 14 c) 10 d) 5

e) 35 f) 7 g) 21 h) 28

i) 15 j) None of these

Solution: c)

18. Suppose $y = 3x - 7$ is an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 1$. Find the values of $f(1)$ and $f'(1)$.

a) $f(1) = -7,\ f'(1) = 3$ b) $f(1) = 3,\ f'(1) = -4$ c) $f(1) = 3,\ f'(1) = -7$

d) $f(1) = -1,\ f'(1) = 3$ e) $f(1) = -4,\ f'(1) = 3$ f) $f(1) = 7,\ f'(1) = 3$

g) Cannot be determined without more information.

Solution: e)

19. Find the derivative $h'(x)$ of the function $h(x) = \dfrac{3e^x + 2x}{\sin x}$.

a) $\dfrac{(3e^x + 2x)\cos x}{\sin^2 x}$ b) $\dfrac{2}{\sin^2 x}$

c) $\dfrac{3xe^{x-1} + 2}{\cos x}$ d) $\dfrac{(3e^x + 2)\sin x}{\sin^2 x}$

e) $\dfrac{(3xe^{x-1} + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$ f) $\dfrac{(3e^x + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$

g) $\dfrac{3e^x + 2}{\cos x}$ h) None of these.

Solution: f)

20. Find $\displaystyle\lim_{x\to 2}\left(\frac{|x-2|}{x-2}\right)$.

a) 0 b) $-\infty$ c) -1 d) 1

e) Does not exist f) -2 g) ∞ h) 2

Solution: e)

Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

21. (8 points) Compute the integrals.

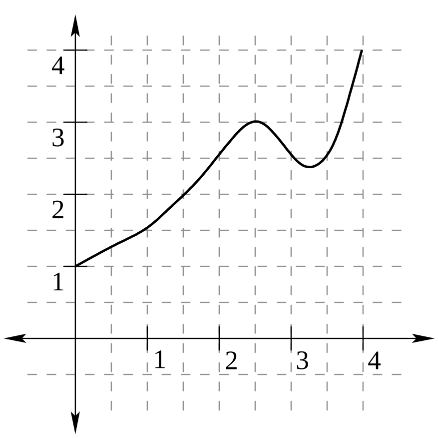
(a) $\displaystyle\int_0^1 \frac{e^{3t}+1}{e^{3t}+3t}\,dt$

(b) $\displaystyle\int_0^2 \frac{x^3+3x^6}{x^6}\,dx$

22. (6 points) The graph of a function f is shown below. Estimate $\displaystyle\int_2^4 f(x)\,dx$ in two ways:

(a) Using four sub-intervals and right endpoints.

(b) Using four sub-intervals and left endpoints.



23. (8 points) A printed poster must have top and bottom margins of 3 inches, and side margins of 2 inches, and the total area must be 600 square inches. What should the dimensions of the poster be to ensure that the printed area is as large as possible?

24. (12 points) Compute the following limits. If the limit is infinite, explain whether the limit is ∞ or $-\infty$ or neither.

   (a) $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{3x^2}$

   (b) $\displaystyle\lim_{x\to -3} \frac{5x}{(x+3)^2}$

   (c) $\displaystyle\lim_{x\to 0} 4x^4 \sin\left(\frac{1}{x}\right)$

25. (5 points) Use implicit differentiation along with the definition of the function $y = \ln(x)$ as the inverse of the exponential function to prove that

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}.$$

   No credit will be given if a method other than implicit differentiation is used.

26. (6 points) Prove that the function $f(x) = 2^x + x$ has at least one root in the interval $(-1, 1)$.

27. (8 points) Gravel is being poured onto the top of a pile that forms a right circular cone in such a way that the radius of the cone is always twice its height. If the gravel is being poured at a rate of 12 m^3/min, at what rate is the height of the cone changing when the height is 3 m? ($V = \frac{1}{3}\pi r^2 h$.)

28. (7 points) Determine where the following function is concave up and concave down on the interval $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

$$f(x) = -e^x \cos(x)$$

END OF EXAM