Math 112 (Calculus I)
Final Exam Form A KEY

Multiple Choice. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

Short Answer. Fill in the blank with the appropriate answer. 1 point each

21. (10 points)

(a) If $f(x) = e^{3x}$ then $f''(x) =$ ____________________

Solution: $9e^{3x}$

(b) $\dfrac{d}{dx}(a^3 + \cos^3 x) =$ ____________________

Solution: $-3\cos^2 x \sin x$

(c) $\dfrac{d}{dx} e^{x^2} =$ ____________________

Solution: $2xe^{x^2}$

(d) $\dfrac{d}{dx}(\tan^{-1}(x^2)) =$ ____________________

Solution: $\dfrac{2x}{1+x^4}$

(e) $\displaystyle\lim_{x\to 0^+} \frac{\ln(1+x)}{x} =$ ____________________

Solution: 0/0 so L'Hôpital gives $\displaystyle\lim_{x\to 0^+} \frac{1/(1+x)}{1} = 1$

(f) $\dfrac{d}{dx} \ln(\sinh(x)) =$ ____________________

Solution: $\dfrac{\cosh(x)}{\sinh(x)}$

(g) $\dfrac{d}{dx} \sin(\pi^2 + e^3) =$ ____________________

Solution: 0

(h) Use the linearization of $f(x) = \sqrt{x}$ at $a = 9$ to approximate $\sqrt{11}$. ____________________

Solution: Linearization at $a = 9$ is $y = \dfrac{1}{6}(x-9) + 3$ so $\sqrt{10} \approx 3 + \dfrac{1}{6} \cdot 2 = 3\dfrac{1}{3}$

(i) $\displaystyle\lim_{x\to 0} \frac{x^2+3}{e^x} =$ ____________________

Solution: 3

(j) $\displaystyle\int 3x^2 + 2x + 1\, dx =$ ____________________

Solution: $x^3 + x^2 + x + C$

Free response: Give your answer in the space provided. Answers not placed in this space will be ignored. 5 points each

22. (6 points) Hannah the Human Cannonball is riding a rocket car at the Bonneville speedway. During her trip, her distance $q(t)$ (measured in kilometers) from the start, t seconds after starting, has been calculated to be given by the function $q(t) = 36t^2 - 2t^3$.

    (a) (3 points) What is her velocity after 1 second?

    Solution: $v(t) = q'(t) = 72t - 6t^2$ so $v(1) = 72 - 6 = \boxed{66\text{m/s}}$.

    (b) (3 points) When does she stop, i.e., at what time $t > 0$ is her velocity zero.?

    Solution: $0 = q'(t) = 72t - 6t^2$ gives $t = 0$s and $\boxed{t = 12\text{s}}$.

23. (8 points) State the definition of the derivative of $f(x)$, and **use the definition** to find the derivative of $f(x) = 3x^2 + 5$.

    Solution:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{3(x+h)^2 + 5 - (3x^2 + 5)}{h} \\ &= \lim_{h \to 0} \frac{6xh + h^2}{h} \\ &= \boxed{6x} \end{aligned}$$

24. (6 points) Compute the definite integral $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{x\,dx}{\sqrt{1 - x^4}}$

    Solution: Let $u = x^2$ to get $\displaystyle\int_{u=0}^{u=\frac{1}{2}} \frac{du/2}{\sqrt{1 - u^2}} = \text{Arcsin}(u)\Big|_0^{1/2} = \boxed{\pi/6}$

25. (6 points) I want to make a fenced rectangular play space for my children, with an area of 96 square meters. One side (the front) will face the road. Most of the fencing will cost me \$10 per meter, but the fencing for the front side costs \$20 per meter because it looks nicer. What should the dimensions be to minimize the total cost of the fencing? How much will it cost?

    Solution:

$$\begin{aligned} Cost &= 10\ell + 20\ell + 10w + 10w = 30\ell + 20w \\ 96 &= Area = \ell w \text{ so } \ell = 96/w \\ Cost &= \frac{30 \cdot 96}{w} + 20w \\ Cost'(w) &= -30 \cdot 96 \cdot w^{-2} + 20 = 0 \\ w^2 &= \frac{30 \cdot 96}{20} = 3 \cdot 48 \\ w &= 12m \end{aligned}$$

$\boxed{w = 12 \qquad \ell = 8 \qquad Cost = \$480}$

Check $Cost'(w) < 0$ for $w \in (0, 12)$ and $Cost'(w) > 0$ for $w \in (12, \infty)$ so this is the absolute minimum.

26. (6 points) If a car is initially moving at 14 meters per second, and the driver begins braking so that the car slows down at the constant rate of 2 meters per second per second until it stops, how far will the car travel from the time that the driver began braking until the time the car stops?

Solution:

$$\begin{aligned} a(t) &= -2m/s^2 \\ v(t) &= -2t + v_0 = -2t + 14 \\ &\text{Stops when } v(t) = 0 \\ t &= 7 \\ Dist &= \int_0^7 v(t)\,dt = -t^2 + 14t\Big|_0^7 = \boxed{49m} \end{aligned}$$

27. (6 points) $\lim_{x\to 0^+} x\csc(x) =$

28. (6 points) Find the derivative of $f(x) = x^{\sin x}$.

29. (6 points) Sketch the graph of a function $f(x)$ which is twice differentiable on the interval $(-\infty, \infty)$ and which has the following properties:

- The second derivative $f''(x)$ is positive on the interval $(0, \infty)$ and negative on the interval $(-\infty, 0)$.
- $\lim_{x\to\infty} f(x) = 2$
- The first derivative is zero only at $x = -3$

Solution: Graph must be

- concave down on the interval $(-\infty, 0)$
- have a max (absolute) at $x = -3$
- have a point of inflection at $x = 0$
- concave up on the interval $(0, \infty)$
- Right hand horizontal asymptote (approached from above) at $y = 2$