Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 113 (Calculus II)
Final Exam

Dec 18, 7:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 10 to 17, show all your work in the space provided.. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is alloted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned
1	10	
MC	24	
10	8	
11	8	
12	6	
13	6	
Sub	62	

#	Possible	Earned
14	6	
15	8	
16a	6	
16b	6	
16c	6	
17	6	
Sub	38	
Total	100	

Short Answer. Fill in the blank with the appropriate answer.

1. (10 points)

   a. Let $y = f(x)$ for $x \in [a, b]$. Give the formula for the length of the curve formed by the graph of this function. _____

   b. Find the vertex of the graph of $y = x^2 + 4x$. _____

   c. The equation $2x^2 + 3x + y^2 - y = 7$ is called a _____.

   d. If $r = f(\theta), \theta \in [a, b]$ is an equation of a curve in polar coordinates, give the formula for the area enclosed by this curve. _____

   e. What does the ratio test predict with regard to the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$? _____

   f. What is the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{4^n}{n} x^n$? _____

   g. The integral $\int_0^{\infty} \frac{dx}{1+x^2}$ equals_____

   h. Here is an antiderivative: $\int \frac{\sqrt{9-x^2}}{x^2} dx$. Tell what substitution to use in order to find this antiderivative._____

   i. The integral $\int_0^1 \frac{1}{x^{2/3}} dx$ equals _____

   j. The antiderivative $\int x \sin(x)\, dx$ equals_____

Multiple Choice. In the grid below fill in the correct answer to each question.

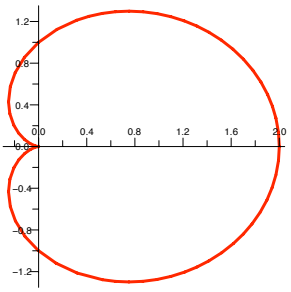
2	A	B	C	D	E	F	G	H	I	J
3	A	B	C	D	E	F	G	H	I	J
4	A	B	C	D	E	F	G	H	I	J
5	A	B	C	D	E	F	G	H	I	J
6	A	B	C	D	E	F	G	H	I	J
7	A	B	C	D	E	F	G	H	I	J
8	A	B	C	D	E	F	G	H	I	J
9	A	B	C	D	E	F	G	H	I	J

2. Find the area enclosed by the polar curve $r = 2 + \sin\theta$ for $\theta \in [0, 2\pi]$.

a) $\frac{5}{2}\pi$ b) $\frac{11}{3}\pi$ c) 2π

d) 4π e) $\frac{9}{2}\pi$ f) None of the above.

3. Identify the equation which goes with the polar graph,



a) $r = 1 + 2\cos\theta$ b) $r = 1 + \cos\theta$ c) $r = 2 + \sin\theta$

d) $r = 2\sin(2\theta)$ e) $r = 2\cos(2\theta)$ f) None of the above.

4. Which of the following series has as its interval of convergence $(-\infty, 1]$?

a) $\sum x^2/n!$ b) $\sum e^{-n}x^{2n}$ c) $\sum \frac{(x-1)^n}{2^n}$

d) $\sum x^{n-1}/n^2$ e) $\sum (x+1)^{n+1}/n^n$ f) None of these

TURN PAGE OVER

5. Determine whether the following series is convergent. If it is, find its sum.

$$\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)} \right)$$

a) It is divergent. b) e c) $\frac{1}{e}$

d) $e - e^{1/2}$ e) $\ln(1)$ f) None of these.

6. If we differentiate the power series

$$\frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!} + ...$$

the resulting function is

a) e^{-x} b) $\cos 2x$ c) $\sin x$

d) only expressible as a series. e) $-(\cos x - 1)$ f) $\cosh x + 1$

7. Find the area between $y = 2x$ and $y = x^2$ for $x \in [-1, 3]$.

a) $\frac{8}{3}$ b) $\frac{4}{3}$ c) $-\frac{4}{3}$

d) 3 e) 4 f) None of the above

8. Let A denote the region between the graphs of $y = \cos(x) + 1$ and the line $y = 1$ for $x \in [0, \pi/2]$. A solid is obtained by revolving A about the x axis. find the volume of the solid.

a) $\frac{\pi^2}{4} + 2\pi$ b) $\frac{\pi^2}{2} + 2\pi$ c) $\frac{\pi^2}{2}$

d) $\frac{\pi^2}{4}$ e) $\frac{\pi^2}{4} - 1$ f) None of these

9. Which of the following integrals represents the surface area of the surface generated by revolving the curve $y = e^{2x}$ for $x \in [0, 1]$ about the line $y = -1$?

a) $\int_0^1 2\pi \left(e^{2x} - 1\right) \sqrt{1 + 4e^{4x}} dx$ b) $\int_0^1 \left(e^{2x} + 1\right) dx$ c) $\int_0^1 2\pi \left(e^{2x} + 1\right) \left(1 + 2e^{2x}\right) dx$

d) $\int_0^1 2\pi \left(e^{2x} + 1\right) \sqrt{1 + e^{4x}} dx$ e) $\int_0^1 \left(e^{4x} + 1\right) dx$ f) None of the above.

Free Response. For problems 10 - 17, write your answers in the space provided. Use the back of the page if needed, indicating that fact. Neatly show all work.

10. (8 points) A ball of radius 5 has a hole of radius 3 drilled completely through it such that the axis of the hole is a diameter of the ball. What is the volume of what is left?

11. (8 points) A spherical tank having radius 10 feet is filled with a fluid which weighs 100 pounds per cubic foot. This tank is half full. Find the work in foot pounds needed to pump the fluid out of a hole in the top of the tank.

12. (6 points) Determine whether the following series converges and explain your answer.

$$\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$$

13. (6 points) The economic trickle-down effect is based on the idea that injection of money into the economy reaches far beyond the initial receiver of funds. Say that one person receives a dollar and spends 4/5 of it. The person who receives that portion spends 4/5 of it in turn and so on and on "forever." The total cash flow from the first dollar is the sum of all these passed-on funds including the first dollar. What is that sum?

14. (6 points) Compute the first 4 nonzero terms of the MacLaurin series for the function $f(x) = \ln(x^2 + 1)$.

15. (8 points) A cube with 2 foot long sides is sitting on the bottom of an aquarium in which the water is 5 feet deep. Find the hydrostatic force on one of the sides of the cube. Use 62.5 pounds per cubic foot as the weight density of water.

16. Find the following

(a) (6 points) $\int_0^{\pi/2} \sin^3(x)\cos^3(x)\,dx$

(b) (6 points) $\int \frac{x+5}{x^2-1}dx$.

(c) (6 points) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

17. (6 points) Here is a parameterized curve called a cycloid. find the equation of the tangent line when $\theta = \pi/3$.

$$x(\theta) = \theta - \sin\theta$$
$$y(\theta) = 1 - \cos(\theta)$$

END OF EXAM