Name:________________________________

Student ID:________________________________

Section:________________________________

Instructor:________________________________

Math 113 (Calculus II)
Final Exam – Form A – Fall 2012 RED

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are not allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned
MC	39	
14	6	
15	6	
16	6	
17	6	
18	6	
Sub	69	

#	Possible	Earned
19	6	
20	6	
21	7	
22	6	
23	6	
Sub	31	
Total	100	

Part I: Multiple Choice *Mark the correct answer on the bubble sheet provided.*

1. Which of the following series converge absolutely?

$$1)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \qquad 2)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad 3)\ \sum_{n=1}^{\infty} \frac{1}{n^3}$$

a) None	b) 1	c) 2
d) 3	e) 1, 2	f) 1, 3
g) 2, 3	h) 1, 2, 3	

2. Find the radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!(x-2)^n}{n^2 3^n}$.

a) 0	b) 1	c) 2
d) 3	e) 2/3	f) 4/3
g) 1/3	h) -2	i) -3
j) ∞		

3. What is the coefficient of x^{100} in the Maclaurin series of e^{-3x^2}?

a) 0	b) 3^{100}	c) $\dfrac{3^{100}}{100}$
d) $-\dfrac{3^{100}}{100!}$	e) $-\dfrac{3^{25}}{25!}$	f) $\dfrac{3^{50}}{50}$
g) $\dfrac{3^{50}}{50!}$	h) $-\dfrac{3^{50}}{50!}$	i) Diverges

4. When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work (measured in foot-pounds) is done in moving it from $x = 1$ to $x = 3$?

a) 6	b) 9	c) 50/3
d) 17/3	e) $-16/3$	f) $-45/2$
g) 27/2	h) 15/2	i) 0

5. The region between the curve $y = \dfrac{1}{x^p}$ and the x-axis for $0 < x \leq 1$ is rotated about the x-axis to form a solid of revolution. For which positive values of p does this solid have *finite* volume?

a) $0 < p$	b) $0 < p < 1$	c) $1 < p$
d) $0 < p < 1/2$	e) $1/2 < p$	f) $0 < p < 2$
g) $2 < p$	h) The volume is infinite for all positive p.	

6. Evaluate the integral $\displaystyle\int_0^{\pi/2} \sin^3(x)\cos^2(x)\,dx$.

a) $1/15$	b) $2/15$	c) $3/2$
d) $1/3$	e) $3/5$	f) π
g) $\pi/3$	h) $\pi/2$	i) 0

7. Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+4x^2}$.

a) 0	b) 1	c) $\arcsin(1/2)$
d) π	e) 2π	f) $\pi/2$
g) $\arctan(\pi)$	h) $\sqrt{2}$	i) $\dfrac{1}{\sqrt{2}}$

8. Which integral represents the length of the curve $y = \sin x + \cos x$, $0 \le x \le \pi/4$? (You might need to set up an integral and do a short calculation.)

a) $\displaystyle\int_0^{\pi/4} \sqrt{2+2\cos x\sin x}\,dx$	b) $\displaystyle\int_0^{\pi/4} \sqrt{2-2\cos x\sin x}\,dx$
c) $\displaystyle\int_0^{\pi/4} \sqrt{2-\cos x\sin x}\,dx$	d) $\displaystyle\int_0^{\pi/4} \sqrt{1-2\cos x\sin x}\,dx$
e) $\displaystyle\int_0^{\pi/4} \sqrt{1-\cos x\sin x}\,dx$	f) $\displaystyle\int_0^{\pi/4} \sqrt{1+\cos x\sin x}\,dx$

9. Which integral represents the area of the surface obtained by rotating the curve

$$y = e^x, \quad 1 \le y \le 8$$

about the y-axis.

a) $\displaystyle\int_1^{8} 2\pi x\sqrt{1+e^{2x}}\,dx$	b) $\displaystyle\int_0^{\ln 8} 2\pi\sqrt{1+e^{2x}}\,dx$
c) $\displaystyle\int_0^{\ln 8} 2\pi x\sqrt{1+e^{2x}}\,dx$	d) $\displaystyle\int_0^{\ln 8} 2\pi e^x\sqrt{1+e^{2x}}\,dx$
e) $\displaystyle\int_1^{8} 2\pi e^x\sqrt{1+e^{x}}\,dx$	f) $\displaystyle\int_1^{8} 2\pi e^x\sqrt{1+e^{2x}}\,dx$

10. If $(\bar{x}, \bar{y})$ is the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$, what is $\bar{y}$?

a) 0	b) $1/2$	c) $1/3$
d) $2/3$	e) $1/4$	f) $3/4$
g) $1/5$	h) $2/5$	i) $3/5$

11. A curve is parametrized by the equations $x = 6\sin t$ and $y = t^2 + t$. Find the slope of the line that is tangent to this curve at the point $(0, 0)$.

a) 0	b) 1	c) 1/2
d) 2	e) 1/3	f) 3
g) 1/6	h) 6	i) Undefined

12. Determine the exact value of the geometric alternating series:

$$\frac{3}{7} - \frac{3}{7^2} + \frac{3}{7^3} - \frac{3}{7^4} + \cdots$$

a) 1/2	b) 3/4	c) 7/6
d) 7/4	e) 3/8	f) 7/8

13. Which of the following three tests will establish that the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$ converges?

1) Comparison Test with $\displaystyle\sum_{n=1}^{\infty} 2n^{-2}$
2) Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} n^{-2}$
3) Comparison Test with $\displaystyle\sum_{n=1}^{\infty} 3n^{-2}$

a) None	b) 1	c) 2
d) 3	e) 1, 2	f) 1, 3
g) 2, 3	h) 1, 2, 3	

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Part II: Written Response *Neatly write the solution to each problem. Complete explanations are required for full credit.*

14. (6 points) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the vertical line $x = 2$.

15. (6 points) Evaluate $\int x \sin(3x)\, dx$.

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16. (6 points) Evaluate $\int \sqrt{1-4x^2}\,dx$.

17. (6 points) Evaluate the integral $\displaystyle\int_1^{\infty} \frac{dx}{x^2+x}$.

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18. (6 points) Let $s(n) = \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$. Find a large enough value of n such that $s(n) \geq 20$, and justify why this choice of n is large enough.

    Hint: Think about the geometric reasoning used in the proof of the Integral Test.

19. (6 points) Determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$.

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20. (6 points) Assuming $0 < x < 1$, evaluate the definite integral $\displaystyle\int_0^x \frac{du}{1+u^7}$ as a power series. Express the answer using summation notation.

21. (7 points) Find the Taylor series for the function $f(x) = \sqrt{x}$ centered at the value $a = 1$. Express the answer using summation notation.

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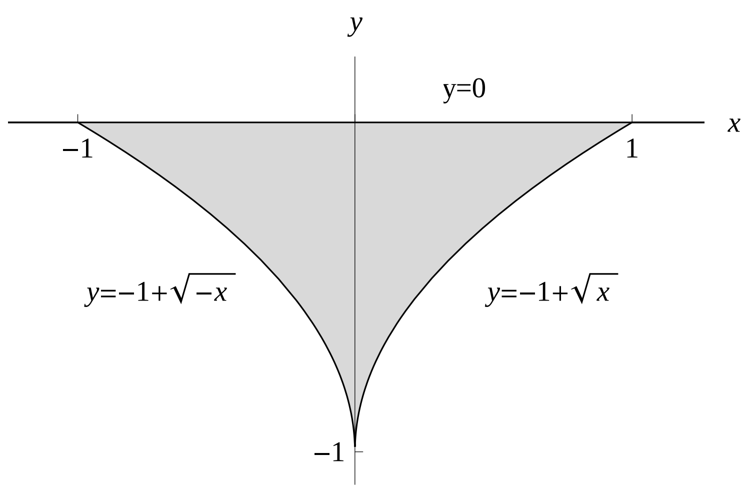
22. (6 points) Find the area of the region that lies inside the first curve and outside the second curve:

$$r = 3\cos\theta, \qquad r = 1 + \cos\theta.$$

23. (6 points) A trough is full of water. Its end is shaped like the shaded region in the picture. The boundaries of the region are the curves $y = 0$ and $y = -1 + |x|^{1/2}$ for $-1 \leq x \leq 1$. If the pressure at depth d is $P = \delta d$, where δ is a constant and d is measured in meters, set up a definite integral for the hydrostatic force F against the end of the trough.

[Note: Set up an integral for F, but don't evaluate the integral. The answer will involve δ.]



END OF EXAM