Math 113 (Calculus II)
Final Exam Form A
April 17, 2009 at 7:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 10 to 18, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as \( \ln(1) \), \( e^0 \), \( \sin(\pi/2) \), etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

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Multiple Choice. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. Find the area of the region enclosed by \( y = x \) and \( y = 5x - x^2 \).
   a) \( \frac{5}{3} \)    b) \( \frac{8}{3} \)    c) \( \frac{16}{3} \)
   d) \( \frac{28}{3} \)    e) \( \frac{32}{3} \)    f) \( \frac{80}{3} \)

2. Set up the integral representing the volume of the solid obtained by rotating the region bounded by \( y = x^2 + 1 \) and \( y = 3 - x^2 \) about the \( x \)-axis.
   a) \( \int_{-1}^{1} \pi [(3 - x^2)^2 - (x^2 + 1)^2] \, dx \)
   b) \( \int_{-1}^{1} 2\pi x [(3 - x^2) - (x^2 + 1)] \, dx \)
   c) \( \int_{-1}^{1} \pi [(x^2 + 1)^2 - (3 - x^2)^2] \, dx \)
   d) \( \int_{-\sqrt{2}}^{\sqrt{2}} 2\pi x [(x^2 + 1) - (3 - x^2)] \, dx \)
   e) \( \int_{-\sqrt{2}}^{\sqrt{2}} \pi [(x^2 + 1)^2 - (3 - x^2)^2] \, dx \)
   f) none of the above

3. Evaluate \( \int_{0}^{\frac{\pi}{2}} \sin^5 x \cos^3 x \, dx \).
   a) \( \frac{1}{24} \)    b) \( \frac{1}{6} \)    c) \( \frac{1}{8} \)
   d) \( -\frac{1}{8} \)    e) \( -\frac{1}{6} \)    f) \( -\frac{1}{24} \)

4. Determine whether \( \int_{0}^{\infty} \frac{1}{1 + x^2} \, dx \) is convergent or divergent. If convergent, evaluate the integral.
   a) divergent    b) 0, convergent    c) \( \frac{\pi}{4} \), convergent
   d) \( \frac{\pi}{2} \), convergent    e) \( \pi \), convergent    f) \( 2\pi \), convergent

5. Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve \( y = e^{2x} \), \( 0 \leq x \leq 1 \) about the \( x \)-axis.
   a) \( \int_{0}^{1} 2\pi x \sqrt{1 + e^{4x}} \, dx \)
   b) \( \int_{0}^{1} 2\pi x \sqrt{1 + 2e^{2x}} \, dx \)
   c) \( \int_{0}^{1} 2\pi x \sqrt{1 + 4e^{4x}} \, dx \)
   d) \( \int_{0}^{1} 2\pi e^{2x} \sqrt{1 + e^{4x}} \, dx \)
   e) \( \int_{0}^{1} 2\pi e^{2x} \sqrt{1 + 2e^{2x}} \, dx \)
   f) \( \int_{0}^{1} 2\pi e^{2x} \sqrt{1 + 4e^{4x}} \, dx \)
6. Find the sum of \( \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} \).

a) \( \frac{1}{28} \)  

b) \( \frac{1}{7} \)  

c) \( \frac{1}{4} \)  

d) \( \frac{4}{7} \)  

e) \( \frac{7}{4} \)  

7. What is the interval of convergence for \( \sum_{n=0}^{\infty} \frac{x^n}{2^n} \)?

a) \( (-\frac{1}{2}, \frac{1}{2}) \)  

b) \( [-\frac{1}{2}, \frac{1}{2}) \)  

c) \( [-\frac{1}{2}, \frac{1}{2}] \)  

d) \( (-2, 2) \)  

e) \( [-2, 2) \)  

f) \( [-2, 2] \)  

8. Find the first 4 terms of the power series for \( f(x) = e^{-x^2} \) centered at 0.

a) \( 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 \)  

b) \( 1 - x + x^2 - x^3 \)  

c) \( 1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6 \)  

d) \( 1 - x^2 + x^4 - x^6 \)  

e) \( 1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 \)  

f) \( 1 + x^2 + x^4 + x^6 \)  

9. Which of the following is the graph of \( r = 3 \cos \theta \)?

a)  

b)  

c)  

d)  

e)  

f)
Free response: Give your answer in the space provided. Answers not placed in this space will be ignored.

10. (7 points) Find the volume of the solid obtained by rotating the region bounded by $x = 1 + (y - 2)^2$ and $x = 2$ about the $x$-axis.

11. (7 points) Evaluate $\int_{0}^{\frac{\pi}{2}} x^2 \sin x \, dx$. 
12. (7 points) Evaluate \( \int \frac{x^3}{\sqrt{x^2 + 1}} \, dx. \)

13. (7 points) Evaluate \( \int \frac{dx}{x^3 - 2x^2 + x}. \)
14. (7 points) Show whether \( \int_{-1}^{2} \frac{1}{x^4} \, dx \) is convergent or divergent. If convergent, evaluate the integral.

15. (7 points) A vertical plate is submerged in water and has the shape shown in the figure. Find the hydrostatic force against one side of the plate. (Use \( \rho g \) to represent the weight density of water.)
16. (7 points) Determine whether the series \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^{2n}}{n!} \) converges absolutely, conditionally or diverges. State which test(s) you use.

17. (7 points) Find the Taylor series for \( f(x) = x^{-2} \) centered at \( a = 1 \).

18. (7 points) Find the area enclosed by \( r = 3 + 2 \sin \theta \).